

# Polarization of Light and Rotation of the Polarization

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## 1 Background and History

In the first of these two experiments you will demonstrate that rotating a piece of apparatus about the direction of a light beam can modify the beam without changing its direction of propagation. That statement evidently can be rephrased as saying that since light is sensitive to rotations of apparatus about its direction of propagation, it must have properties transverse to its direction of propagation – That it has transverse polarization. While the work of Maxwell<sup>1</sup> and Hertz<sup>2</sup> laid the framework for our present understanding of light as a transverse electromagnetic wave, many properties such as transverse polarization and phenomenological details of the interaction of light with matter had been established by earlier work. Notable among these was the law of Malus<sup>3</sup> which showed that the intensity of light transmitted through two polarizing substances is proportional to the square of the cosine of the angle between the orientations of the substances. A more helpful restatement is that the amplitude of the wave transmitted is proportional to the cosine of the angle between the axis of the polarizer and the polarization of the incident wave.

In the first experiment you will verify the law of Malus for several polarizers. In the second you will observe the phenomena of optical activity, the property of some substances to rotate the plane of polarization of light transmitted through it. Many substances which do not show optical activity in the absence of a magnetic field, show an analogous effect, called the Faraday<sup>4</sup> effect if a magnetic field is present in a direction parallel or anti parallel to the direction of the light.

## 2 First Experiment: Linear Polarization and Polarizers

### 2.1 apparatus

Dolan Jenner Fiber Lite light source  
adjustable circular aperture stop (diaphragm)  
cadmium-sulfide cell and multimeter with computer readout  
3 linear polarizers  
optical bench to mount the above

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<sup>1</sup>James Clerk Maxwell, 1831-1879, Scottish physicist who combined electricity and magnetism into one subject which predicted electromagnetic waves which happened to have the same speed as light. He also did important work in thermodynamics and statistical mechanics.

<sup>2</sup>Heinrich Hertz, 1857-1894, German Scientist, did the experimental work which showed that sparks from high voltage discharge produce (radio) waves which have the properties Maxwell described: the same speed as light and can be reflected, refracted and diffracted. Our unit of frequency, the hertz, is named after him.

<sup>3</sup>Etienne Louis Malus, 1775-1812, French physicist and mathematician, discovered the law which bears his name and also discovered polarization by reflection and double refraction in crystals.

<sup>4</sup>Michael Faraday, 1791-1867, English Physicist and Chemist, considered by many to have been the greatest experimentalist who ever lived. His contributions include Faraday's Law of Induction, the concept of magnetic lines of force, and the quantitative relationship between mass deposited and charge transfer in electrolysis as well as the optical rotation described here.

## 2.2 Procedure

The process of obtaining linearly polarized light from an unpolarized source and the attenuation of linearly polarized light by subsequent polarizers are almost trivial consequences of the linearity of Maxwell's Equations and the fact that the electric field is a vector. Linearity implies that we can add or decompose into components the amplitudes of light waves just as for any vectors. If we figure out what happens to the electric field of the light wave, the magnetic field will do what ever is required to maintain the wave relationship between the electric and magnetic fields. The intensity of the wave (in a vacuum) is just proportional to the square of the electric field amplitude.

For an unpolarized source the electric vector of the emitted wave must have the same magnitude in any direction perpendicular to the direction of propagation of the wave, so we cannot simply apply the law of Malus for the first polaroid. If we choose the z axis in the direction of propagation of the unpolarized wave, then the x and y components of the electric field are equal. Moreover this is true however we choose to rotate our x and y directions about the z axis!! A polarizer picks out one of these possible x or y directions and transmits only the part of the wave which has its electric field in that direction. Cutting out one of these components reduces the intensity by a factor of two.

The photo cell used to make measurements of the transmission of polaroids in this experiment is a Cadmium Sulfoselenide (CdS) Photoconductive Photocell. The intensity of light is inversely proportional to the 1.5 power of the resistance[1] for R greater than a few hundred Ohms. You will be interested in the ratios of intensities of light. Thus if  $I_r$  and  $R_r$  are the reference intensity and resistance, a second pair of values is related to the reference values as  $I/I_r = (R_r/R)^{1.5}$ .

1. Position the source at one end of the optical bench and the detector facing the source about 12 inches away. Put the aperture stop (diaphragm) about two inches in front of the source and make sure that the transmitted illumination is symmetric about the photocell. Leave positions for subsequent mounting of polarizers between the stop and detector. Note the resistance of the detector. If the resistance is less than about 500  $\Omega$ , the intensity is too high, the simple 1.5 power rule is not correct, and you need to reduce the intensity. This intensity will be used as a comparison with intensities in the next parts of the experiment. It depends upon the distance from the source, so do not move either the source or detector during the course of the experiment. We will define our own units of illumination, defining the level at the photocell to be one in this configuration. By comparing all resistances to this resistance, the subsequent intensities will be given in these units. (We might define this illumination to be one Malus and then all subsequent measurement would be fractions of a Malus.)
2. Measure the background light – the light which gets to the detector without going down the beam axis. Cover the hole in the diaphragm plate with an opaque object and record the reading of the ohm meter. This is the background light. Do not put paper on the table under the optical bench where it can scatter light into the detector. Calculate the brightness of the background light. This brightness (NOT the resistance) should be subtracted from all subsequent light measurements. If you change the distance from source to aperture, the background light scattered from the table and walls will change and this correction will need to be remeasured.
3. Place one of the polarizers in the slot closest to the source. Be sure it is centered in the beam so that light to the detector goes through the polaroid and is not blocked by the shield. Note the intensity and whether it changes as the polarizer is rotated in its mount about the direction of the light. If there is a measurable change, note the angles and resistances at maximum and minimum intensity. Use the resistance of the photocell to calculate the ratio of intensity with the polaroid to that without the polaroid. Comment on why it is less than half.
4. Place a second polarizer in the position nearest the detector and centered on the beam. Rotate the polarizer near the detector to locate the positions of maximum transmission. Measure the resistance as the polaroid is rotated in  $10^\circ$  steps over a range which includes two maxima and extends to where there is a clear drop beyond the maxima. Plot your intensities *vs.* the (subtracted) polaroid angle. Plot  $\cos^2\theta$  on the same graph. Explain why you expect the two curves to be similar. The component of the transmitted field goes as the first power of  $\cos\theta$ , so where did that second power come from?

5. Estimate how precisely you could determine the angle of the second polaroid for minimum transmission. You will use a clever experimental technique to locate this minimum in the second half of this experiment and compare the precision of the two methods.
6. Consider placing a third polarizer between the two presently on the bench. Is it “obvious” to you that light which is not stopped by the first is stopped by the second, so no light will get through no matter what you place in between? Try it and see. If you observe any transmission develop an explanation. An acceptable explanation involves a derivation of an equation to describe the intensity as a function of the rotation angle of the middle polaroid. Measure a few points on the rotation curve of the middle polaroid so that you can make a qualitative check of your theory.

## 2.3 Extension to Many Polaroids

1. Suppose you had  $N$  polaroids to place after the first (a total of  $N+1$  polaroids), that the second was rotated  $90^\circ/N$  from the first, the third rotated  $90^\circ/N$  from the second, *etc.*, so that the last was rotated  $90^\circ$  from the first. What polarization and what fraction of the intensity with no polarizers present would be transmitted?
2. Note that  $\cos \theta \sim 1 - \theta^2/2$ ,  $(1 - x)^m \sim 1 - mx$  for  $\theta$ ,  $x \ll 1$  and repeat the previous exercise taking the limit for  $N$  very large. Congratulations! You have just discovered optical activity or at least one way of visualizing it. Thus you are prepared for the second part of this study of polarized light.

# 3 Second Experiment: Optical Activity – Rotation of the Plane of Polarization

## 3.1 Theory

You saw in the previous section how the plane of polarization of a linearly polarized wave could be rotated by a large number of sheets of polaroids. Early workers visualized optical activity as due to rotation by planes of molecules. More quantitative calculations based on the quantum nature of energy levels relate the rotation to different speeds of right and left circularly polarized components of the beam. A linearly polarized beam can be analyzed as the sum of right and left circularly polarized (helicity or chirality) components. The speed of a wave in material depends upon the energy difference between the photon energy and absorption lines in the material. The absorption lines depend upon the energy levels of the electrons in the material, and angular momentum selection rules only allow coupling of specific helicity photons to specific angular momentum states of the electrons (Zeeman effect for the case with a magnetic field). Thus the absorption and speed are different for the two circular polarizations.

The effect with no magnetic field occurs for molecules or crystals with a helical structure. If only one helicity molecule is present, light with the helicity which is the same as that of the molecules will have a different speed from that of the opposite helicity, and the plane of polarization of a linearly polarized wave will rotate as it propagates through the substance. The example of sugar, which is used here, is particularly interesting. The sugar molecule is helical and the symmetry of the electromagnetic field, which provides the forces that hold the molecule together, is symmetric under the parity (mirror) reflection which turns right hand helices (screw threads) into left handed helices. Electrodynamics therefore predicts that right and left handed molecules should occur with equal abundances and consequently the effect should average out for sugar molecules in solution. If sugar is produced by artificial chemical synthesis, that is indeed the result. However most commercial sugar (sucrose) is produced by sugar cane or sugar beets, and all such sugars have the right handed form. The biochemistry of the two forms is quite different; left handed sugar does taste sweet but has no nutritional benefits (for us - great for diets but prohibitively expensive). Apparently the earliest bacterial life happened to form with right handed molecules, and thus all subsequent life, including starch and sugar producing plants as well as us, has inherited that helicity.

The rotations in laboratory quantities of optically active materials tend to be small so that sensitive apparatus is needed to measure the effect. We need much better methods of locating the direction of

polarization than we used in the previous section and that has led to the development of polarimeters which are based upon comparing intensities in adjacent parts of an image. The two sides of a beam of light are polarized at slightly different angles and viewed through a third polarizer. When the two halves of the beam have equal intensity, the line dividing them disappears, and the eyepiece polarizer is then oriented half way between the planes of polarization of the beam. At low intensity, the eye is extremely sensitive to intensity differences in such adjacent images, and the position of equal intensity can be located to a few tenths of a degree. If an optically active substance is placed between the polarizer and the analyzer, both beam polarizations will be rotated and the balance position will be rotated the same amount.

A sodium vapor lamp is used for the optical activity measurements because the emission is primarily in a close doublet with wavelengths 589.592 and 588.995 nm. Much of the interaction of light with matter depends upon the wavelength, so the nearly monochromatic light from the sodium lamp produces cleaner results than would be obtained with a broad spectrum source.

### 3.2 Apparatus

1. Light source, Sodium (close doublet, 589.6 and 589.0 nm)
2. American Krueger and Toll Company (distributors) Polarimeter
3. 3 tubes to hold liquids in the polarimeter – See Appendix A for background on the the length of the tubes. One is unmarked, one is marked F and contains a sugar (Kroger brand) solution, and the third, marked splenda, contains a solution of a sugar substitute.
4. Solenoid, See Appendix A for solenoid parameters.
5. Power supply, Instek SPS1820 18V/20A (20 Amp)

### 3.3 Procedure

1. Procedural notes
  - (a) The sodium vapor lamp requires about 15 minutes to warm up, so turn it on immediately when you start the experiment and do not turn it off.
  - (b) The polarimeter readout is a vernier with a coarse readout of  $1/4$  degree and a vernier scale marked in 0.01 degree intervals.  $0^\circ = 360^\circ$  is in the region you will be reading. Treat  $1^\circ$  as  $361^\circ$  rather than using negative values for angles on the other side of zero because the direction of labels on the vernier would have to be reversed for use with negative angles.
  - (c) This is a high precision device and you will be measuring angles with a precision of a few hundredths of a degree. Even though this looks like a pretty stiff device, mechanical distortions (twists) of a few hundredths of a degree are possible. In particular, do not over tighten the two thumb screws which hold the solenoid or the tubes in place. The screws are not perfectly aligned and over tightening them twists the eyepiece relative to the incident polarizer.
  - (d) The two halves of the beam are split by optical elements adjusted by the lever under the source end. It rotates the plane of only one of the beams, so the split moves by half of the adjustment. It is not a precision adjustment, so it must not be changed between precision measurements which are to be compared.
  - (e) The eye is very sensitive to slight differences in brightness in dim light but much less sensitive in brighter light. Be sure to make your measurements at a dark rotation angle.
2. Use the large vernier to measure the length of one of the tubes, including the end caps. Measure the distance from end of the cap to the outside window surface with the depth gauge on the small vernier. Open the tube containing water, remove the window from the cap and measure the thickness of the window. Use these measurements to determine the length the of the liquid in the assembled tube and compare with Bob Patchen's measurement in the appendix of this write up.

3. With no material between the collimator and eyepiece, focus the eyepiece so that the line dividing two halves of the field of view is sharp. Do this by looking into the center eyepiece and rotating the eyepiece, which is on a coarse thread. The split image you see is due to the slightly different polarizations of light on the two sides. The angle between these polarizations is set by the lever under the lens near the source. Set this lever at  $5^\circ$ , find an angular region where both sides are dim, and measure the angle of equal brightness. The analyzer angle is measured on an angular scale equipped with a vernier and read through the magnifiers at either side of the eyepiece. The mirrors reflect light from the source to illuminate the scale. If you are not accustomed to reading verniers, experiment with the linear vernier caliper left with your equipment and ask the instructor for assistance. You may want to measure the angle at which each side appears to blend in with the other and average the two readings. Note that the angular region of interest is where the polaroids in the eyepiece are approximately orthogonal to those in the collimator. These dark regions occur at approximately  $0^\circ$  and  $180^\circ$  on the angle scale with regions of maximum brightness at right angles. Both partners should make several such measurements and then repeat with the scale at the front set at  $10^\circ$ . You may wish to try  $2^\circ$  as well. The point of this exercise is to find the split which enables you to locate the equal brightness angle (called balance point in this description) with the best reproducibility. The optimum setting depends upon the material in the light path. For, example for a saturated sugar solution, the light scattering from one path to the other is so bad that the  $2^\circ$  setting is useless and moving up to  $10^\circ$  or even  $15^\circ$  is necessary. Note that when you changed from  $5^\circ$  to  $10^\circ$  the reading of the analyzer polaroid changed by only about  $2.5^\circ$ . Why? As part of your comparison of reproducibility, you should estimate the error on the angle of the balance position and use it to assign an error to the Verdet constant and specific rotation which you measure in the next two parts of this experiment.
4. Measure the Faraday effect (Verdet constant) for water. The Verdet constant is the rotation per unit length per unit magnetic field. (Minutes for the rotation, cm for path length in the field and Gauss for the field are common units. A Gauss is  $10^{-4}$  Tesla. In these units for water,  $V = 0.0131 \text{ min G}^{-1} \text{ cm}^{-1}$  [2]). First evaluate contribution of the windows at the end of the tube to the rotation. Place the an empty tube inside the solenoid and attach the solenoid in place of the hinged tube between the polarizer and analyzer. Use one of the plastic spacers at each end of the tube and be careful that the tube does not slide out of the solenoid as you maneuver the solenoid into place in the polarimeter. Measure the balance position with no current and with 20 A in the solenoid. Fill the tube with water, refocus the eyepiece if necessary, and measure the balance position. Think about the choices for the split of the analyzer and choose the setting which is most reproducible. Turn on the current to the solenoid and measure the balance position with currents of 5, 10, 15, and 20 amperes. The coil will overheat if left at 20 Amp for more than about 10 minutes, so the current should be turned down while reading the angle.. Is the rotation proportional to the field? What value (and error) do you obtain for Verdet's constant for water from these 5 measurements?
5. Measure the optical activity of two concentrations of sugar and of splenda. Put the tube marked F in the plain carrier (not the solenoid) and measure the balance position. You may wish to use a different split. Scattering of light from one beam into the other is worse at higher concentrations, so pick a split which gives a good measurement with the F solution. You want to compare rotations of water with the three solutions, so do not change the split during this series of measurements. The unlabeled tube was filled with water for the Faraday effect measurements. You will have to empty it for the empty measurement and then refill it from the F/2 bottle which is sugar at half the concentration of the F tube. This series of measurements should be F, water, F/2, Splenda. Check the rotation with no tube present. You may have to refocus for this measurement.

The rotation is expected to be proportional to the number of sugar molecules in the beam, which means it is proportional to the product of the number of sugar molecules per unit volume times the length of the beam in the solution. The units of specific rotation are traditionally degrees per gram of optically active component (sugar here) per cc of solution per 10 cm path length, and in these units, the specific rotation of sucrose (common sugar from either beets or cane) is 66 [3]. Use the specific rotation given above and compute the concentrations of the two sugar solutions. Also

compare the rotation of water with no liquid to determine if the water contributes to the rotation. As a point of comparison, a saturated sugar solution would have about 450 g of sugar in 500 ml of solution, much higher than the concentrations used here.

## 4 Appendix A — Update of parameters, February 2007 and 2008

Students have been getting great results with these two experiments and we were concerned that our values for the parameters of the tubes and solenoid did not match the quality of the student work. Moreover some alert students noticed points we had missed, so we reviewed and adjusted the parameters you are to use in reducing your measurements.

### 4.1 Length of the tubes

The lengths of the brass tubes is 23.90 cm, but there is a gasket between the window and the brass on each end. The gaskets are 3.5 mm thick but are compressed when the caps are tightened enough to prevent leaks. The lip on the end caps interferes with direct measurement with standard machine shop calipers. Bob Patchen from the Science Shop assisted with the measurements and suggested putting a 3/8 inch ball bearing against each window and measuring the length including the ball bearings and then subtracting the diameters of the two bearings and the two window thicknesses. This method gave 24.37 cm with the caps tight enough to avoid leaks. When you change liquids, turn the caps snug but do not over tighten them or the liquid length will be less than this value.

### 4.2 The Solenoid

The solenoid was made by Bob Patchin in our machine shop. It is 6 layers, 80 turns/layer of # 10 AWG (30 Amp max) wire on a spool. The length of the winding is  $L = 23.42$  cm. The inner diameter of the winding is 3.492 cm and the outer diameter is 6.46 cm; it is a long solenoid but not a very long solenoid. For a very long solenoid the magnetic field inside the coil far from the ends is given by  $B_o = \mu_o(N/L)I = 2.576 \times 10^{-3} I$  Tesla for this coil. The current is  $I$  and  $(N/L) = 480/0.2342$  is the total number of turns over the length of the coil. For a finite length solenoid, the field at the midpoint is decreased by a factor  $\cos \theta$  where  $\theta$  is the angle from the mid point to the coil at the end. Since this is a thick solenoid, the calculation is done by dividing the coil into 6 layers and the correction is averaged, giving a correction factor of 0.9776 or  $B_M/I = 0.9776 B_o/I = 2.518$  mT/Amp. The field at the center is not explicitly used in this application, but a cross check of measurements at the center with a high precision Hall Probe gave  $B_M = 2.6028$  mT/Amp with a rmsd of 0.0006, 3.2% higher than the computed value. The calibration of the high precision probe is long out of date, so it is conceivable that the probe is in error. Further checks against the field of the pair of coils used in the Hall Probe experiment led to further inconsistencies. Additional work is needed and we recommend that you multiply the field given by the calculation in this note by 1.015 and assign it an error of 1.5%.

The axial field decreases near the ends of the coil and the tube is slightly longer than the coil. The Faraday rotation depends upon the integral of the magnetic field over the length of the liquid in the tube. The field on the axis of the coil at a distance  $z$  from the middle is:

$$B(z) = \frac{\mu_o NI}{4b} \left( \frac{b-z}{(r^2 + (b-z)^2)^{1/2}} + \frac{b+z}{(r^2 + (b+z)^2)^{1/2}} \right)$$

where  $b$  is the half length of the coil and  $r$  is the radius of the thin coil. Integrating from  $-\ell$  to  $\ell$ , over the length of the tube of liquid, gives

$$\int_{-\ell}^{\ell} B(z) dz = B_o \left( (r^2 + (\ell + b)^2)^{1/2} - (r^2 + (\ell - b)^2)^{1/2} \right)$$

and averaging over the six layers gives

$$\int_{-\ell}^{\ell} B(z)dz = 0.2150B_oI = 0.5537 \times 10^{-3}I \text{ T} \cdot \text{m}.$$

The preceding calculation is accurate to a few percent but students have been measuring rotation angles well enough that they pushed us to do better on the magnetic field calculation and to justify using the integral along the centerline of the coil. Specific points at which the preceding calculation may be inadequate are:

1. The field decreases near the ends of the solenoid. The tube is about 1.5 cm shorter than the space, so it may be displaced 0.75 cm from the symmetric position assumed above. If the fall off is linear, the gain on one end is compensated by loss at the other, so we need to investigate the curvature of the field decrease near the end of the tube. Plastic tubes 0.75 cm long have been made to put into the solenoid at the ends of the tube, reducing the slop in the tube position by more than a factor of 10.
2. The winding is a spiral or helix and the calculation was based on a plane circle of current. The current has a longitudinal as well as azimuthal component. The longitudinal component is reduced by the cosine of the pitch angle. There are 80 turns in the 23.4 cm length of the coil and the inner layer has a radius of 1.6 cm. The circumference is 10 cm. The pitch is  $(23.4/80)/10 = 29$  mrad. The cosine of this angle is 0.9996, and makes an insignificant decrease in the axial field. The longitudinal component of the current produces an axial field proportional to the sine of this angle, but the six layers are wound with alternating pitches and cancel in pairs to a good approximation. Inside the coil, the transverse fields from the longitudinal component on opposite sides of the coil are in opposite directions. Good cancellation of small effects.
3. The light beam has a finite diameter. What is the variation of the field across the beam. The beam aperture is limited by the 1 cm hole at the eyepiece end of the sample space. The physical aperture is larger at the opposite end, but the diameter of the light spot is about 1.2 cm. After exploring several series approximations I concluded that the easiest way to calculate the field off axis was to start with the Biot-Savart Law for a single loop and then add the contributions of the individual turns.

The Biot-Savart law gives the magnetic field at the field point  $\mathbf{x}$  as the integral over all space produced by a current density  $\mathbf{J}(\mathbf{x}')$  where  $\mathbf{x}'$  is the 3-dimensional integration variable:

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_o}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}') \times (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} d^3x'$$

To do the integral, pick a coordinate system with the coil in the xy plane and the z axis through the center of the loop. (Caution: We will integrate vectors. Since it is only in Cartesian coordinates that the unit vectors are constant, we need to use Cartesian components, but the coefficients of the unit vectors are most conveniently put into cylindrical coordinates for this problem.) Since the problem has symmetry about the z axis, we can, with no loss of generality, choose a coordinate system in which the field point is on the x axis. In this coordinate system the y-component of the field is zero at the field point we have selected and the x-component in Cartesian coordinates is equal to the radial component. Then the field point is  $\mathbf{x} = \rho\mathbf{i} + z\mathbf{k}$  and, since the integrand is zero unless  $\rho' = a$  and  $z' = 0$ , the integration variable is  $\mathbf{x}' = a\cos\phi'\mathbf{i} + a\sin\phi'\mathbf{j}$ , and  $\mathbf{J} = -I\cos\phi'\mathbf{i} + I\sin\phi'\mathbf{j}$ . Substituting, working out the cross product and making minor trig simplifications:

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_o Ia}{4\pi} \int_{-\pi}^{\pi} \frac{z\cos\phi'\mathbf{i} + z\sin\phi'\mathbf{j} + (a - \rho\cos\phi')\mathbf{k}}{(\rho^2 + a^2 + z^2 - 2a\rho\cos\phi')^{3/2}} d\phi'$$

Since our unit vectors are Cartesian and do not depend upon  $\phi'$ , the components can be split up and the unit vectors taken outside the integrals. By symmetry, the field can have only radial (x with our choice of coordinate system) and z components. The equation shows that the y component is given by an odd

function,  $\sin\phi'$ , over and even function, which integrates to zero. The field of the solenoid is obtained by numerical integration of this equation and summing over the turns. An additional integration over the length of the tube gives the result needed to compute the expected rotation of the plane of polarization. The radial component will have opposite signs at the ends of the solenoid, so it will tend to cancel for light rays which do not cross the axis. The integral of the radial field is much smaller than the integral of the axial field in the region of interest and it does not rotate the plane of polarization, so it will not be discussed further here. The results of the numerical integrations are presented in the following tables and plots. Table 1 shows that the variation of the axial field as a function of radius in the mid plane of the coil is amazingly small, too small to show at four figures over the region of illumination of our tube and only 5 at in the fourth figure at a radius of 1.5 cm, out almost to the inner coil.

Table 1: Mid coil Field variation

radius, cm	0.00	0.60	1.00	1.50
B/B <sub>o</sub> at z=0	0.9776	0.9777	0.9778	0.9781

Table 2 shows the effective length of the tube as a function of the radial displacement of the line of integration and of the longitudinal displacement of the tube. The integral of the field is the long solenoid central field,  $B_o = \mu_o(N/L)I = 2.574 \times 10^{-3} I$  Tesla, multiplied by the appropriate one of these effective lengths.

Table 2: Effective Length of Tube

radius, cm	0.00	0.60	1.00	1.50
offset, cm	effective lengths in m			
0.00	0.2150	0.2153	0.2160	0.2173
0.25	0.2148	0.2152	0.2158	0.2171
0.50	0.2145	0.2148	0.2154	0.2167
0.75	0.2139	0.2142	0.2148	0.2159
1.00	0.2131	0.2134	0.2139	0.2149

Note that the variation due to misplacing the tube along the axis by 0.75 cm is half a percent so it is clearly useful to use the plastic spacers which hold the tube position to within about a tenth that. The change with radius is 4 parts in 2000 or 0.2%.

Plots of the axial component of the field for several radial displacements are shown in Figure 1 and more detailed blow-ups of the field at the center and end are shown in Figures 2 and 3.

The three plots show the axial field as a function of distance from the mid plane of the solenoid. The two solid curves are on axis and 6 mm off axis, the region of the light paths. The two solid curves are so nearly identical that greater detail near the center and near the end of the coil is shown on the next two figures. Additional curves at 1 cm and 1.5 cm are shown only for comparison and to show how remarkably uniform the field is in a solenoid. These curves give qualitative justification for the numerical comparisons of effective lengths in the table. Note that they are very slightly curved near the end of the tube. A displacement of the tube pushes one end of the tube into higher field and other into lower field. If the curves were straight, the gain and loss would cancel. The curvature is perceptible over the full 1.5 cm of displacement possible with no spacers, but the curvature is negligible over the much smaller range of displacement with the spacers.

The conclusion of this calculation and discussion is that the plastic spacers should be used. The simple calculation of the field along the axis is then adequate:

$$\int_{-\ell}^{\ell} B(z)dz = 0.2150B_oI = 0.5538 \times 10^{-3} I \text{ T} \cdot \text{m}.$$

**EXCEPT** that, pending resolution of the Hall probe calibration problem, this value should be multiplied by 1.015 and assigned a 1.5% error,  $(0.5621 \pm 0.0084) \text{ IT} \cdot \text{m}$ .



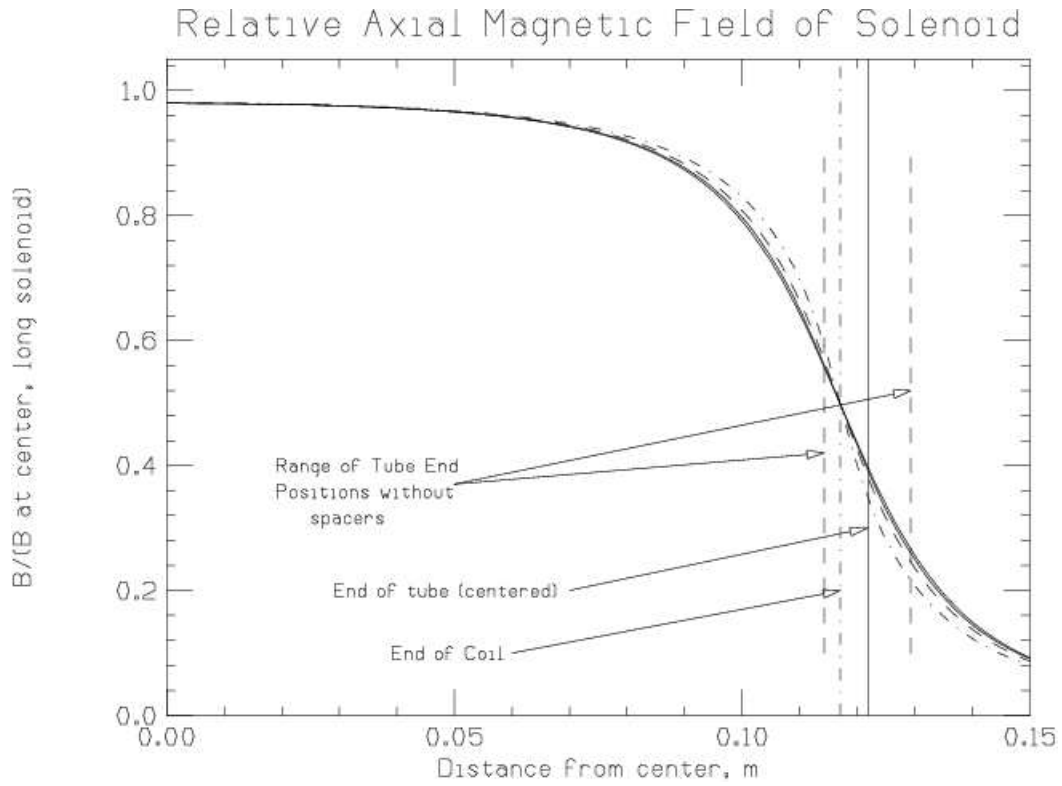


Figure 1: Field *vs.* *z* for several radial displacements. The two solid curves show the axial component of *B* on axis and at a radius of 6 mm and are nearly indistinguishable. The dashed curve is at a radial displacement of one cm and the dot-dash curve is at 1.5 cm, just inside the inner coil. The curves cross at the end of the solenoid and inside the solenoid the field increases very slowly with increasing radius.

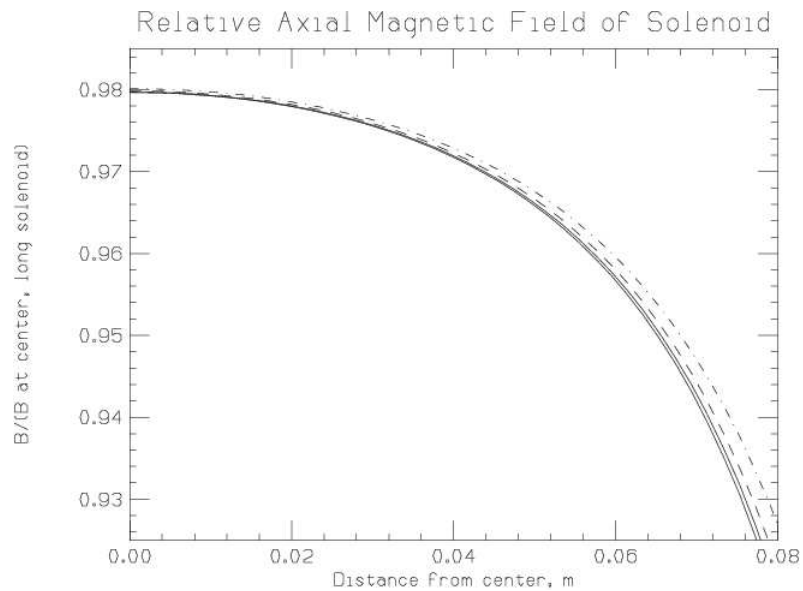


Figure 2: Expanded view of Figure 1 near the middle of the coil. The separation of the two solid curves is much clearer at this higher magnification,

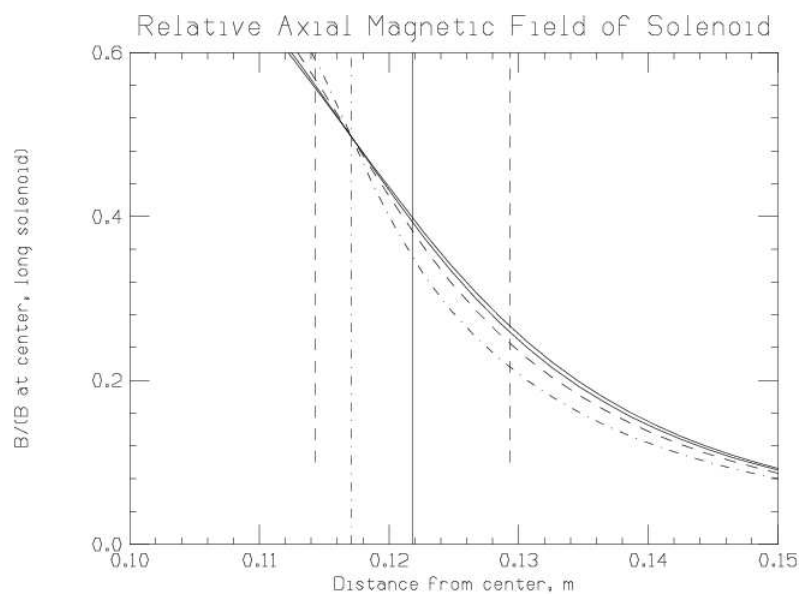


Figure 3: Expanded view of Figure 1 near the end of the coil. Note that the curves all cross at the end of the coil.

## 5 References

1. Photonic Detectors, Inc., Cadmium Sulfoselenide (CdS) Photoconductive Photocells, Type PDV-P500x, purchased in 2003. Data sheets and response curves for a newer model, PDV-P9001, are available (2007) at:  
<http://eecs.oregonstate.edu/education/classes/engr201/files/pdvp9001.pdf>  
or, for a qualitative description of how photoresistors work:  
[http://optoelectronics.globalspec.com/LearnMore/Optics\\_Optical\\_Components/Optoelectronics/Photoconductive\\_Cells](http://optoelectronics.globalspec.com/LearnMore/Optics_Optical_Components/Optoelectronics/Photoconductive_Cells)
2. CRC Handbook of Chemistry and Physics, 50th Ed., 1969-1970, (Chemical Rubber Publishing Co.) or almost any optics text such as Frank L. Pedrotti, S.J. and Leno S. Pedrotti, Introduction to Optics, Second Edition 1993 (Prentice Hall, Englewood Cliffs, NJ 07632)
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