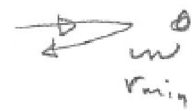
Worksheet 3 solutions:

Consider Rutherford's argument for a compact nucleus. An incoming alpha particle (we know is 2 protons and 2 neutrons) has a head on collision with a gold nucleus (we know is 79 protons and 188 neutrons). If the alpha particle penetrates the nucleus it will likely continue on through, but if it recoils, it is trading all its kinetic energy for potential energy when it bounces back. For an incoming alpha particle with kinetic energy of 7.7 MeV that bounces back, how close did it get to the nucleus? (this is a rough upper limit to the size of the nucleus.) (What happens to the probability of a scatter at a low angle for Rutherford scattering?) (K=8.99e9 Nm²/C²)

$E = KE + PE = \frac{1}{2}mv^2 + \frac{kq_1q_2}{r}$ @ $r=\infty$, $E = KE_0$

so $KE_0 = \frac{kq_1q_2}{r_{min}}$ @ r_{min}, $E = PE$



$r_{min} = \frac{kq_1q_2}{KE_0} = \frac{\overbrace{1.44\,eVnm}^{kq_0^2} \cdot (79)(2)}{7.7\times10^6\,eV}$

$= 2.95\times10^{-5}\,nm$ or $29.5\,fm$

scattering $\propto \frac{1}{\sin^4\theta/2} \rightarrow \infty$ as $\theta \rightarrow 0$ { in practice we don't really see this happen since scattering from the nucleus gets screened by e's }

Positronium:

Let’s continue with this hypothesis that the atomic system is 2 particles, but lets examine the case where an electron and a positron (positive electron) make up the system.

-What force provides the acceleration needed to keep the electron/positron in orbit (assume a circular orbit around the electron/positron center of mass)? Please write an equation for the balance of forces in a circular orbit like this) (Does a stable orbit seem reasonable (classically)? What happens if we take a charge and shake it?) . Please reduce your equation to the coulomb term and the force term involving the reduced mass μ, the distance between the electron and the positron, r, and the orbital frequency, ω.

Ans: Coulomb force! Classically I’d expect the electron to radiate energy and the orbit get smaller and smaller. An accelerating (shaken) charge should emit radiation classically.

For the electron:

$$m_e r_e \omega^2 = k q_e^2 / r^2$$

$$m_e r_e = m_{po} r_{po} \ ; \ r = r_e + r_{po} \ ; \ r = r_e + (m_e / m_{po}) r_e$$

$$\frac{m_{po} r}{m_e + m_{po}} = r_e \quad so \quad m_e r_e = \frac{m_e m_{po} r}{m_e + m_{po}} = \mu r$$

$$\mu r \omega^2 = k q_e^2 / r^2$$

Now consider the total energy of such a system. Write down an equation for the total energy of the system. Now please use your force balance equation to get an expression for the total energy just in terms of the coulomb term and a constant. Don't forget to include both the electron and the positron. Please reduce your equation to the coulomb term and a kinetic energy term involving the reduced mass μ, the distance between the electron and the positron, r, and the orbital frequency, ω.

$$E = (1/2)m_e r_e r_e \omega^2 + (1/2)m_{po} r_{po} r_{po} \omega^2 - kq_e^2 / r = (1/2)m_e r_e r_e \omega^2 + (1/2)m_e r_e r_{po} \omega^2 - kq_e^2 / r$$
$$= (1/2)m_e r_e r \omega^2 - kq_e^2 / r = (1/2)\mu(r\omega)^2 - kq_e^2 / r$$

Now, lets add Bohr's quantization condition.
Use your expression for force balance to find an expression for the radius of the orbit that does not involve the momentum. (what is this value for n=1? Can you do it without looking up anything?)

$$(m_e r_e^2 + m_{po} r_{po}^2)\omega = n\hbar = \mu r^2 \omega$$
$$\mu r \omega^2 = kq_e^2 / r^2 = \mu r (n\hbar / \mu r^2)^2$$
$$r = (n\hbar)^2 / (\mu k q_e^2) = 2(n\hbar)^2 / (m_e k q_e^2) = 2r_o$$

Now use your expression for the radius and the expression for the total energy (with the coloumb term) and find the values of the energy levels. What is the lowest energy level?

$$E = (1/2)\mu(r\omega)^2 - kq_e^2 / r = (1/2)kq_e^2 / r - kq_e^2 / r = -(1/2)kq_e^2 / r$$
$$E = -(1/2)kq_e^2 / r = -(1/2)kq_e^2 (\mu k q_e^2) / (n\hbar)^2 = -(1/4)m_e (kq_e^2)^2 / (n\hbar)^2$$
$$E = -(1/2)13.6eV / n^2 = -6.8eV / n^2$$

Photons are emitted when the electron/positron goes from a higher n to a lower n. What energy photon is emitted going from the n=2 to n=1 state?

$$\Delta E = 6.8eV((1/1^2) - (1/2^2)) = (3/4)6.8eV = 5.1eV$$