Rotation of a Wheel by a Falling Mass

1) A solid wheel has radius R, mass M, and moment of inertia $I = MR^2/2$
2) A light cord is wrapped around the wheel from which mass m hangs
3) What is the angular acceleration α of the wheel?
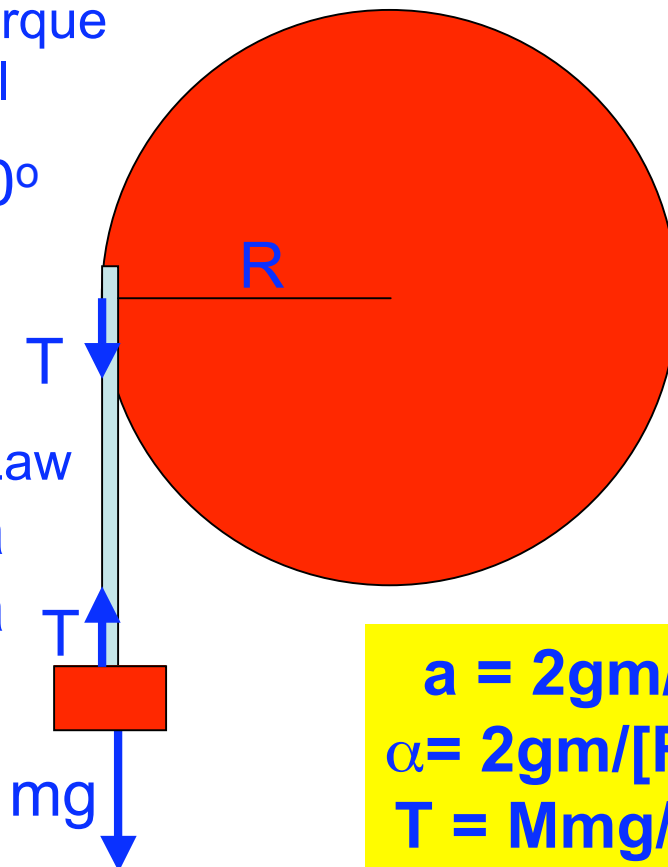4) What is the tension T in the cord?

1) T produces a torque τ on the wheel

$$\tau = R*T*\sin90^o$$
$$\tau = RT$$

2) Newton's 2nd Law
$$mg - T = ma$$
$$T = mg - ma$$

T

T

mg

Note: T < mg

R

3) Torque is equal to moment of inertia times angular acceleration $\tau = I\alpha$

4) Using $\tau = RT$ and using $\alpha = a/R$ we get
$$RT = Ia/R$$

5) $T = Ia/R^2 = Ma/2$
$$mg - ma = Ma/2$$

$$a = 2gm/(M+2m)$$
$$\alpha = 2gm/[R(M+2m)]$$
$$T = Mmg/(M + 2m)$$