Rotational Equilibrium of a Ladder
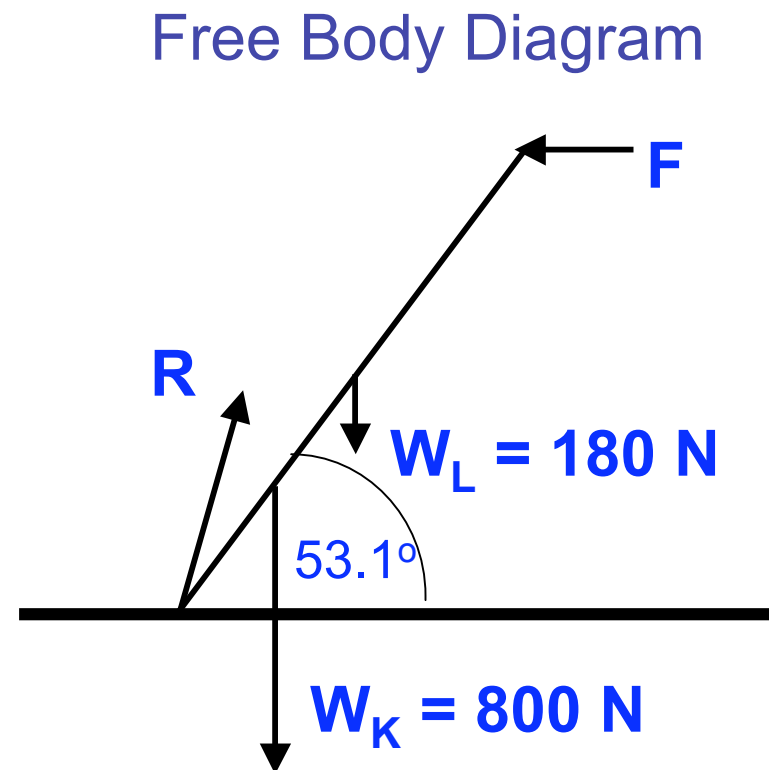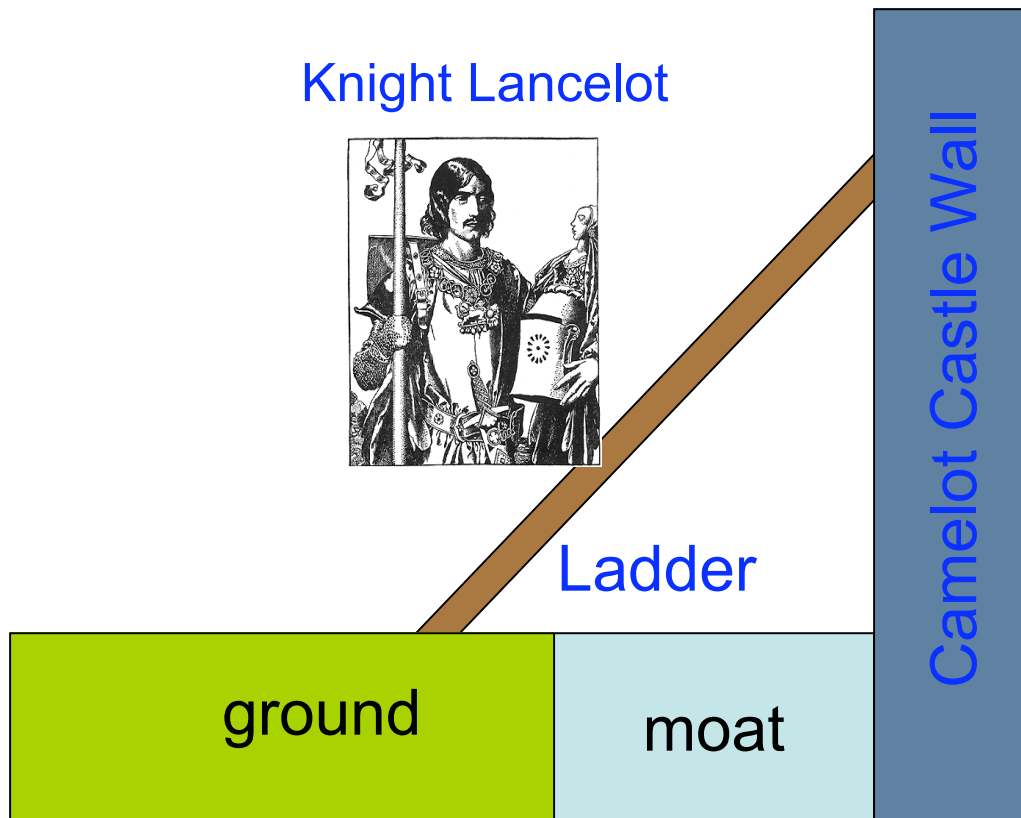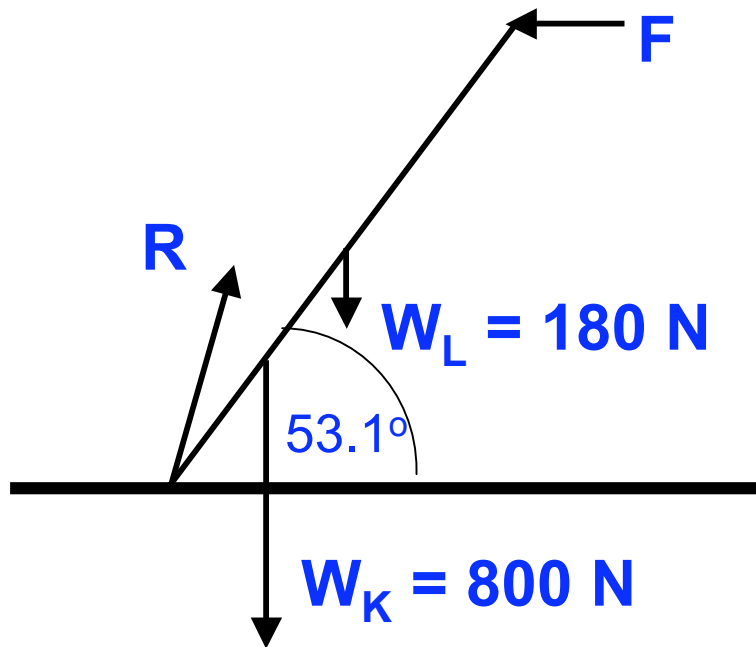
1) A 5 m ladder with weight 180 N leans at 53.1° against a castle wall
2) Lancelot, weighing 800 N, is 1/3 up the ladder. The wall is frictionless.
3) The ground has a static friction coefficient $\mu_s = 0.35$
4) What is the force of the ground **R** on the ladder?
5) What is the force of the wall **F** on the ladder?
6) Can Knight Lancelot ascend all the way to the top of the ladder?

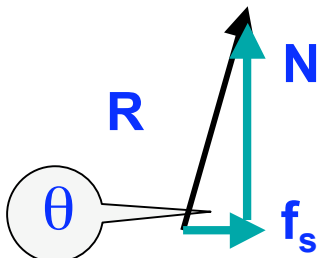Knight Lancelot

Ladder

ground

moat

Camelot Castle Wall

Free Body Diagram

F

R

$W_L = 180\ N$

53.1°

$W_K = 800\ N$

Rotational Equilibrium of a Ladder, 1/3 Ascent

Free Body Diagram

F

R

W_L = 180 N

53.1°

W_K = 800 N

Force from ground F is at an angle θ above the horizontal

$f_s = R\cos\theta$, $N = R\sin\theta$

N

R

θ

f_s

1) $\Sigma F_x = 0 = R\cos\theta - F = f_s - F$

   $\Rightarrow f_s = F$

2) $\Sigma F_y = 0 = R\sin\theta - W_L - W_K$

   $\Rightarrow N = W_L + W_K = 980$ N

Compute torques about bottom of ladder

3) $\Sigma \tau = 0 = F*5*\sin(53.1) - W_L*2.5*\cos(53.1) - W_K*(5/3)*\cos(53.1)$

   $\Rightarrow F = 268$ N $\Rightarrow f_s = 268$ N

4) $R = \sqrt{(268)^2 + (980)^2} = 1020$ N

   $\theta = \text{atan}(980/268) = 75°$

Check on friction

$f_s^{max} = \mu_s*N = 0.35*980 = 343$ N > 268 N

Rotational Equilibrium of a Ladder, Full Ascent

Free Body Diagram
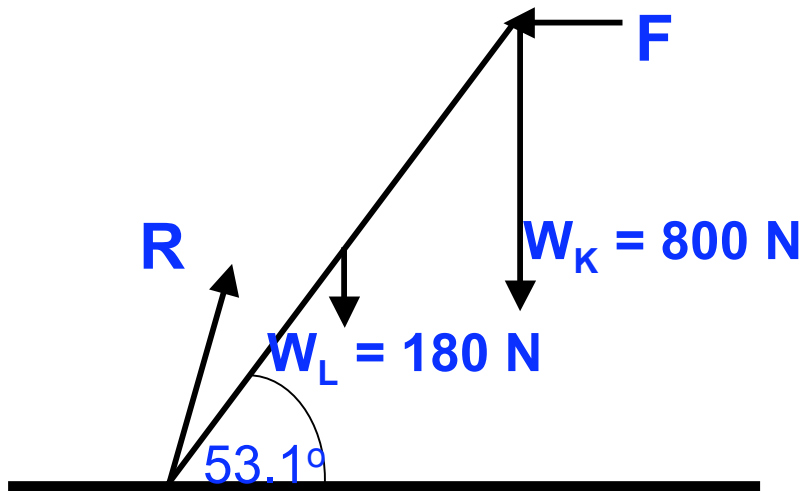
F

$W_K = 800$ N

R

$W_L = 180$ N

53.1°

Force from ground F is at an angle θ above the horizontal
$f_s = R\cos\theta$, $N = R\sin\theta$

1) $\Sigma F_x = 0 = R\cos\theta - F = f_s - F$

$\Longrightarrow f_s = F$
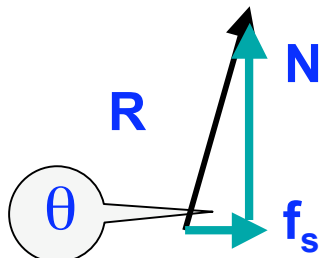
2) $\Sigma F_y = 0 = R\sin\theta - W_L - W_K$

$\Longrightarrow N = W_L + W_K = 980$ N

Compute torques about bottom of ladder

3) $\Sigma\tau = 0 = F*5*\sin(53.1) - W_L*2.5*\cos(53.1) - W_K*(5)*\cos(53.1)$

$\Longrightarrow F = 667$ N $\Longrightarrow f_s = 667$ N

Check on Friction
$f_s^{max} = \mu_s*N = 0.35*980 = 343$ N < 667 N
Ladder will slip before Lancelot gets to top !
Ladders can be dangerous to your health.

N

R

θ

f_s