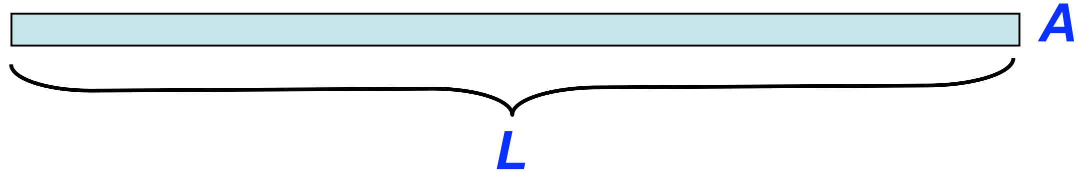
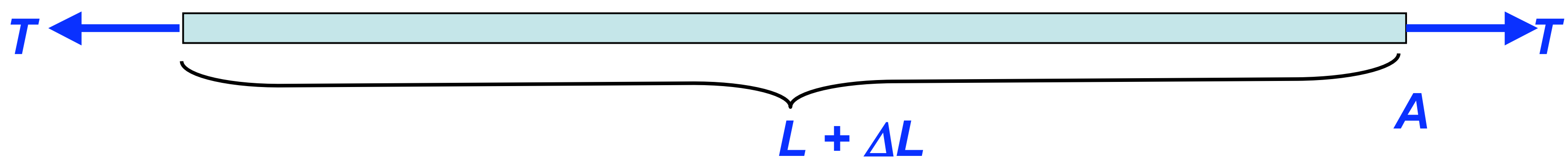
Stress, Strain, and Young's Modulus

An unstretched wire of length ***L*** and Cross-Sectional Area ***A***



A tension ***T*** stretches the wire to length ***L + ΔL*** and Cross-Sectional Area ***A***



Definitions and Equation

1) <u>Stress</u> = **T/A** force/area (N/m^2 or Pa)
2) <u>Strain</u> = ***ΔL/L*** dimensionless ratio
3) Stress/Strain = ***Y*** Elastic Modulus (Pa)

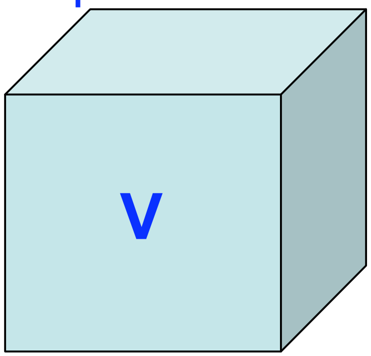
Example: A steel rod has a Young's modulus of 20×10^{10} Pa (1 Pascal = 1 N/m^2)
The rod has a cross sectional area of 0.30 cm^2 and an unstretched length 2.0 m
The rod supports a weight of 550 kg. How much will it stretch?

$(T/A)/(\Delta L/L) = Y \longrightarrow \Delta L = (T/A)*L/Y = (550*9.8/(3.0\times10^{-5}))*2.0/20\times10^{10}$

$\Delta L = 1.8 \times 10^{-3}$ meters

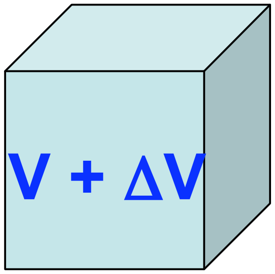
Bulk Stress, Bulk Strain, and Bulk Modulus

An uncompressed volume ***V***



Outside pressure = p

A compressed volume ***V + ΔV***



$\Delta V < 0$

$\Delta p > 0$

Outside pressure = p + Δp

Definitions and Equation

1) Bulk Stress = **Δp** force/area (N/m^2 or Pa)
2) Bulk Strain = ***ΔV/V*** dimensionless ratio
3) Bulk Stress/Bulk Strain = ***B*** Bulk Modulus (Pa)

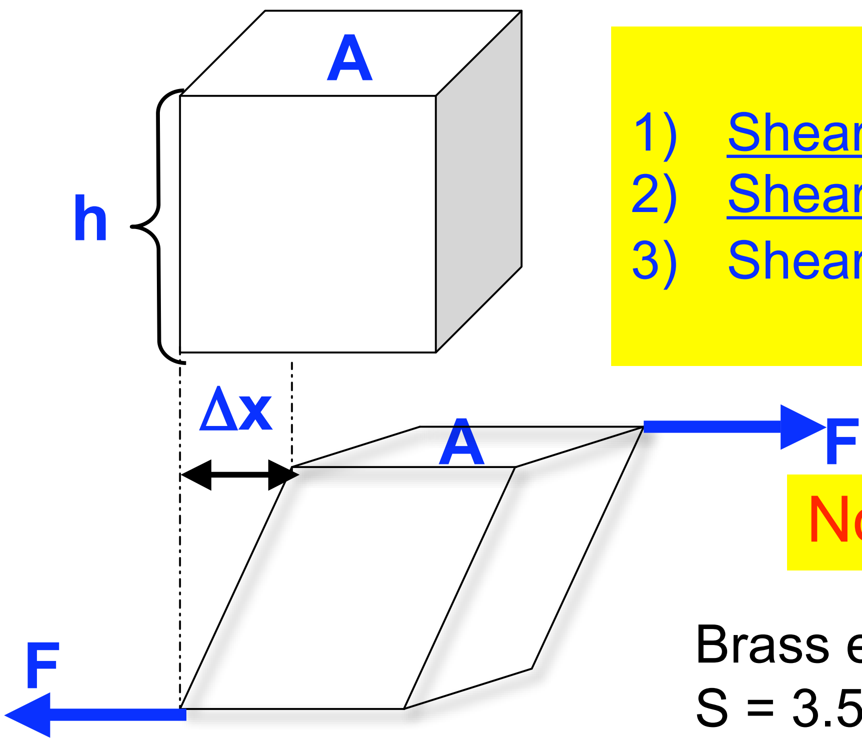
$$B = -\Delta p/(\Delta V/V)$$

Note the negative sign in the definition for **B**

A positive pressure change **Δp** causes a negative **ΔV** change

Shear Stress, Shear Strain, and Shear Modulus

An unstressed volume ***V = hA***



Volume under shear stress

Definitions and Equation

1) Shear Stress = **F/A** force/area (N/m^2 or Pa)
2) Shear Strain = ***Δx/h*** dimensionless ratio
3) Shear Stress/Shear Strain = Shear Modulus

$$S = (F/A)/(\Delta x/h) \quad \text{(Pa)}$$

Note the Δx is almost always << h

Brass example: A = (0.8 m*0.005 m), h = 0.8 m $S = 3.5 \times 10^{10}$ Pa. How much force is needed to obtain a $\Delta x = 1.6 \times 10^{-4}$ m?

Answer: F = S*(Δx/h)*A

$F = 3.5 \times 10^{10}*(1.6 \times 10^{-4}/0.80)*(0.80*0.005)$

$F = 2.8 \times 10^{4}$ N (= ~3 Tons)