Final Exam Key First Sample Final

1. A **Carnot** engine whose high temperature (T_H) source is at 127$^{\text{O}}$C takes in 200 calories and expels 160 calories to the low temperature (T_C) sink. What is the value of the low temperature T_C (answer in $^{\text{O}}$C)

a) 102 b) 87 c) 40 d) 0 e)* none of these (KEY: You must convert to Kelvin)

$$\epsilon = \frac{T_H - T_C}{T_H} = \frac{W}{Q_H} = \frac{40}{200} = 0.20 \Longrightarrow T_C = T_H(1 - 0.20) = 400 \cdot 0.8 = 320 \text{ K} = 47 \text{ C}$$

2. One kilogram of water at 0$^{\text{O}}$C is heated to 100$^{\text{O}}$C, but NOT turned into steam. What is the change in entropy (specific heat of water is 1 cal/(gram-C$^{\text{O}}$); answer in calories/Kelvin)?

a)* 312 b) 20 c) 10 d) 0 e)* none of these

$$\Delta S = \int_{T_1}^{T_2} \frac{dQ}{T} = \int_{273}^{373} \frac{mc\,dT}{T} = 1000 \cdot 1.0 \ln\left(\frac{373}{273}\right) = 312 \text{ calories/Kelvin}$$

3. A coal burning TVA electric power plant produces 800 MW (mega-Watts) of power and has an efficiency of 30%. It loses its waste heat by means of large cooling towers which evaporate water ($L_V = 2256$ kJ/kg). Assuming all of the waste heat goes into evaporating the water, how much water is evaporated in one second (answer in kg)?

a)* 827 b) 2,667 c) 677 d) 2.82 e) none of these

In one second the power plant produces 800 mega-Joules since one Watt = one Joule/sec:

$$\epsilon \equiv \frac{W}{Q_H} = 0.30 \Longrightarrow Q_H = 2,667 \text{ mega-Joules} \Longrightarrow Q_C = 1,867 \text{ mega-Joules}$$

where Q_C is the wasted heat energy in one second. This corresponds to a mass M of vaporized water equal to

$$M = \frac{1,867 \text{ x } 10^6}{2,256 \text{ x } 10^3} = 827 \text{ kg}$$

4. What fraction of an iceberg's volume is submerged ($\rho_S = 1025$ kg/m^3, $\rho_I = 917$ kg/m^3)?

a) 95% b) 93% c) 91% d)* 89% e) none of these

The weight of the iceberg $W_I = \rho_I g V_I$ must be equal to the weight of the water displaced by the submerged part of the volume $W_S = \rho_S g V_S$. This gives

$$\rho_I g V_I = \rho_S g V_S \Longrightarrow \frac{V_S}{V_I} = \frac{\rho_I}{\rho_S} = 0.89$$

5. Two stars of masses M and $6M$ are separated by a distance D. How from from star M should a third mass be placed in order that it have no net gravitational force acting on it?

a) $0.41D$ b) $0.33D$ c) $0.37D$ d)* $0.29D$ e) none of these

See the **Lecture 23** notes for almost the same problem worked out.

6. Two vectors have magnitudes 10 and 11, and their scalar product is -100. What is the vector sum of these two vectors?

a) 6.6 b)* 4.6 c) 8.3 d) 9.8 e) none of these

$$\vec{A}\cdot\vec{B} \equiv AB\cos\theta = 10\cdot 11\cos\theta = -100 \Longrightarrow \theta = -155^{\mathrm{O}}$$

The magnitude of the sum of two vectors is given by the Law of Cosines

$$\vec{C} \equiv \vec{A}+\vec{B} \Longrightarrow C = \sqrt{A^2+B^2+2AB\cos\theta} = \sqrt{100+121-200} = 4.6$$

7. A rock is thrown downward from an unknown height h with an initial speed of 10 m/s. It strikes the ground 3.0 seconds later. What is the value of h (answer in meters)?

a) 44 b) 14 c)* 74 d) 30 e) none of these

$$y(t) = y_0 + v_0 t - \frac{1}{2}gt^2 = 0 = h - 10\cdot 3 - 4.9\cdot 9 \Longrightarrow h = 74 \text{ m}$$

8. A person whose weight is 800 N is riding in an elevator which is accelerating downward at the rate of 1.5 m/s^2. What is the magnitude of the force of the elevator floor on the person (answer in N)?

a)* 680 b) 800 c) 920 d) 120 e) none of these

The mass of the person is 800/9.8 = 82 kg, and the person is experiencing an effective gravitational acceleration = 9.8 - 1.5 = 8.3 m/s^2. So the person experiences an effective weight of $82\cdot 8.3 = 680$ Newtons.

9. A 50 kg child is riding in a Ferris wheel which has a radius of 10 m and travels in a vertical circle. The Ferris wheel completes one revolution every 10 seconds. What is the magnitude of the force on the child exerted by the seat in which the child is sitting when the Ferris wheel is at the top if its motion (answer in N, ignore any forces exerted by a seatbelt or a restraining bar)?

a)* 290 b) 490 c) 690 d) 200 e) none of these

The centripetal force on the child is $mv^2/r = 50\cdot 6.3^2/10 = 200$ N. The weight of the child is 490 N, of which 198 N goes into the centripetal force. That leaves 290 N for the force of the seat on the child.

10. A person lifted a 2.0 kg object from the bottom of a well at a constant speed of 2.0 m/s for 5.0 s. How much work was done (answer in Joules)?

a) 220 b)* 200 c) 240 d) 270 e) none of these

$$W = Fd = wvt = 2.0\cdot 9.8\cdot 2.0\cdot 5 = 200 \text{ Joules}$$

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11. A 1.2 kg mass is projected from ground level at some unknown angle with speed of 30 m/s. The mass is seen to just clear a 16 m high fence before falling back to the ground. What was the kinetic energy of the mass when it cleared the fence (ignore air friction and use conservation of energy, answer in J)?

a)* 352 b) 188 c) 0.0 d) insufficient information provided e) none of these

The initial energy is all kinetic $= 0.5 \cdot mv^2 = 540$ Joules. At the 16 m maximum height there is gravitational potential energy $mgh = 188$ Joules. So the difference is the remaining kinetic energy.

12. A 2.0 kg ball moving with a velocity of $3\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}}$ bounces off the floor such that its new velocity is $3\,\hat{\mathbf{i}}+4\,\hat{\mathbf{j}}$. What is the impulse exerted on the ball by the floor ($\hat{\mathbf{j}}$ is in the vertical direction, answer in Newton-seconds)?

a)* $+16\,\hat{\mathbf{j}}$ b) $-16\,\hat{\mathbf{j}}$ c) $+12\,\hat{\mathbf{i}}$ d) $-12\,\hat{\mathbf{i}}$ e) none of these

$$J \equiv F\Delta t = \Delta p = m(3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}) - m(3\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}}) = 2 \cdot 8\,\hat{\mathbf{j}} = +16\,\hat{\mathbf{j}}$$

13. A horizontal disk with a 10 cm radius rotates about a vertical axis through its center. The disk starts from rest at $t = 0$ seconds and has a constant angular acceleration of 2.1 rad/s^2. At what time t will the centripetal (a_c or a_r) and the tangential (a_t) components of the total acceleration be equal in magnitude (answer in seconds)?

a) 0.55 b) 0.63 c)* 0.69 d) 0.59 e) none of these

$$a_c = \frac{v^2}{r} = r\alpha^2 t^2 = a_r = \alpha r \Longrightarrow t = \sqrt{1/\alpha} = 0.69 \text{ s}$$

14. A merry-go-round has a radius $R = 2$ m and a moment of inertia $I = 250$ kg-m^2. It is rotating about its center at 10 revolutions per minute. A child of mass 25 kg hops onto edge of the merry-go-round. What is the new rotational speed of the merry-go-round with the child at the outer edge (answer in revolutions per minute)?

a) 10 b) 9 c) 8 d)* 7 e) none of these

Use conservation of angular momentum $L = I\omega$. When the child hops on to the merry-go-round, the moment of inertia increases by $25 \cdot r^2 = 100$ kg-m^2, so the new ω_f is $\omega_i \cdot 250/350 = 0.71\omega_i$, or 7 revolutions per second.

15. A particle has its oscillatory motion described by $x(t) = 10 \sin(\pi t + \pi/3)$ where x is in meters and t is in seconds. At what time are the potential and kinetic energies equal (answer in seconds)?

a) 0.7 b) 0.8 c)* 0.9 d) 0.6 e) none of these

The potential energy is given by $k(x(t))^2/2$ and the kinetic energy is given by $m(v(t))^2/2$ where $k = m\omega^2$ and $\omega = \pi$ rad/sec in this problem. So the value of the mass does not matter.

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Part II Worked Problems Solve each of the problems. *Show clearly all your work and which equations you use.* Partial credit will be given. All numerical answers must have units attached where appropriate. (15 points each. Spend no more than 10 minutes/problem.)

1. In the figure shown a 225 kg mass is suspended from a strut which makes a $45^{\rm O}$ angle with the horizontal. In turn, the strut is clamped to the floor with a hinge and also supported by a cable at $30^{\rm O}$ to the horizontal which has a tension T. The strut is 3.0 m long and has a mass of 45.0 kg.

a) Making use of the above figure, draw a free-body diagram showing all the forces acting on the strut.

There is a weight force W_m of the mass acting straight down at the upper end of the strut. There is a weight force W_s acting down at the middle of the strut. There is the tension force T acting along the cable. There is the reaction force R of the hinge acting at some unknown angle θ (up and to the right).

b) Write down *three* independent equations indicating that the strut is in rotational and translational equilibrium.

$$\text{HORIZONTAL TRANSLATIONAL EQUILIBRIUM} \quad R\cos\theta = T\cos 30$$

$$\text{VERTICAL TRANSLATIONAL EQUILIBRIUM} \quad R\sin\theta = T\sin 30 + W_S + W_m$$

$$\text{ROTATIONAL EQUILIBRIUM} \quad T\cdot 3.0\sin 15 = W_s\cdot 1.5\cos 45 + W_m\cdot 3.0\cos 45$$

The torques are computed about the hinge point because that is where two of the unknown forces act. The $15^{\rm O}$ angle is the angle between the line of action of T and the radius vector from the hinge to the point of application of T.

c) Solve for the tension T

From the torque equation alone one gets $T = 6630$ N.

d) Solve for the horizontal and the vertical components of the force R which is begin exerted by the hinge on the strut.

Substitute the value of T into the two equations for horizontal and vertical equilibrium. This will give $R_v = 5960$ N, and $R_h = 5740$ N.

2. In the figure an ideal *diatomic* gas is caused to pass through three branches making a closed cycle as shown. From point 1 to point 2 there is an isothermal expansion. From point 2 to point 3 there is an isochoric drop in pressure, and from point 3 back to point 1 there is an adiabatic compression. The volume at points 2 and 3 is triple the volume V_1 at the original point 1, and you may take the pressure at point 1 to be P_1 and the temperature at point 1 to be T_1.

a) In terms of the original point 1 values P_1, T_1, and V_1, what are the values of P_2 and T_2 for point 2.

Along the isothermal part of the cycle, one can use $P_1V_1 = P_2V_2$. Of course $T_2 = T_1$ and $V_2 = 3.0V_1$ was given. This gives $P_2 = P_1/3$.

b) In terms of the original point 1 values P_1, T_1, and V_1, what are the values of P_3 and T_3 for point 3.

Along the adiabatic part of the cycle, one can use $P_1V_1^\gamma = P_2V_2^\gamma$ where $\gamma = 1.41$ for a diatomic gas. This gives $P_3 = P_1 \cdot (0.3)^{1.4} = 0.19P_1$. The volume at point 3 is given as $V_3 = 3.0V_1$, so one can solve for the temperature at point 3 using the ideal gas law $PV = nRT$ so

$$\frac{P_1V_1}{T_1} = \frac{P_3V_3}{T_3} \Longrightarrow T_3 = \frac{P_3V_3}{P_1V_1}T_1 = 0.64T_1$$

c) What is the ΔQ and the ΔE_{int} for each of the three branches of the closed cycle?

For the isothermal part of the cycle, the heat added is the work done which is given by $W = nRT\ln(V_2/V_1)$. There is not change in internal energy in an isothermal process.

In the isochoric part of the process, the heat added is $nC_V\Delta T = nC_V(T_3 - T_2) = nC_V(T_3 - T_1)$, and no work is done.

In the adiabatic part of the process, no heat is added and the change in internal energy is equal to the negative of the work done. The change in internal energy is equal to $nC_V\Delta T$ since internal energy depends only on the change in temperature.

d) What is the net work done in the closed cycle, where again your answer is in terms of P_1, T_1, and V_1?

These were given in the previous answer.

3. A 5.00 kg block is pulled along a horizontal frictionless table as shown in the figure. The force F is 12.0 N acting at 25^{O} above the horizontal.

a) What is the acceleration of the block?

The horizontal component of the force accelerates the block while the vertical component of the force just decreases the normal force of the table. So

$$F_h = F\cos(25) = 12 \cdot 10.9 = ma = 5.0 \cdot a \Longrightarrow = 2.2 \text{ m/s}^2$$

.

b) Suppose the force F is increased slowly, but still acts at 25^{O} above the horizontal. What is the maximum value of the magnitude of F consistent with the block remaining on the table?

The maximum force would be such that the vertical component of F would be equal to the weight force:

$$F_v = F\sin(25) = w \Longrightarrow F = \frac{5 \cdot 9.8}{\sin 25} = 116 \text{ n}$$

c) What is the acceleration of the block for the force F given in part b) ?

Repeat the solution for part a) except use the 116 N value from part b). This will give $a = 21.0$ m/s^2.

4. A mass of 1.30 kg is placed at the end of a light rod 0.780 m long having negligible mass. This system rotates in a horizontal circle about the other end of the rod at 5010 revolutions/minute.

a) What is the moment of inertia of this system about the axis of rotation?

The moment of inertia is simply $mr^2 = 1.30 \cdot (0.780)^2 = 0.79$ kg-m^2.

b) Air resistance exerts a force of 0.023 Newtons on the mass, opposite to the direction of motion at all times. How much torque must be applied to the system in order to keep it rotating at constant angular velocity?

The air resistance creates a torque which will slow down and stop the motion, so an opposite sized torque must be applied to keep the motion going.

$$\tau_{\text{air}} = Fr = 0.023 \cdot 0.780 = 0.018 \text{ N-m}$$

5. The density of steel is 7800 kg/m^3, and the maximum stress that can be placed on steel cable is 7.0 x 10^8 N/m^2. What is the fastest speed of a transverse wave in a steel cable? (Note your answer should not depend on the diameter or thickness of the cable.)

For a solid material the speed of a transverse wave is given by

$$v = \sqrt{F/\mu}$$

where μ is the linear mass density, and F is the tension. The value of F is the product of the stress (force/area) and the area:

$$F = SA$$

The linear mass density is the total mass of the cable, given by density times volume, divided by the length. For a long cable (cylinder) the volume is just the area times the length. So one has:

$$\mu \equiv \frac{M}{L} = \frac{\rho A L}{L} = \rho A$$

This gives the speed of the wave as

$$v = \sqrt{\frac{SA}{\rho A}} = \sqrt{\frac{S}{\rho}}$$

Substituting in the numbers you should obtain $v = 300$ m/s.

6 In the diagram there is a thick aluminum cylinder 85.0 cm long. A steel wire, also 85.0 cm long, is attached to the aluminum cylinder by means of fasteners on each end of the cylinder. The whole system is at 10.0^{O} C initially, as in the diagram, and then it is heated to 120^{O} C.

a) What is the length expansion of the big aluminum cylinder (the coefficient of linear expansion for aluminum is 23 x $10^{-6}/\mathrm{C}^{\mathrm{O}}$)?

The change in the length of the aluminum is given by

$$\Delta L_A = L\alpha_A \Delta T = 85.0 \cdot 23 \cdot 10^{-6} \cdot 110 = 0.215 \text{ cm}$$

b) What *would have been* the length expansion of the steel wire (the coefficient of linear expansion for steel is 11 x $10^{-6}/\mathrm{C}^{\mathrm{O}}$)?

The change in the length of the steel would be given by

$$\Delta L_S = L\alpha_S \Delta T = 85.0 \cdot \cdot 10^{-6} \cdot 110 = 0.103 \text{ cm}$$

c) Because it is attached to the aluminum cylinder, the steel wire actually stretches the same length as did the aluminum in part a), so there is a tension force in the steel because of the extra stretching. What is the size of that tension force assuming that the steel wire had a diameter of 0.1 cm originally (The *Elastic* or *Young's* Modulus for steel is $E = 10^9$ N/m^2)?

The steel is stretched an extra 0.112 cm because it is attached to the aluminum. This extra stretch generates a tension, and a stress which is given according to the Young's modulus relationship:

$$\frac{\Delta L_S}{L} = \frac{T/A}{E}$$

Using the area of a 0.1 cm diameter wire for A, you should compute T as

$$T = 10^9 \cdot \frac{0.112}{85}\pi \cdot (0.0005)^2 = 1.03 \text{ N}$$

Actually the Young's modulus for steel is 200 x 10^9 N/m^2, so the true value would be more like 200 N.