Answer key for 116a final exam, Spring 2008

Part I Multiple Choice, 5 points each

1. Almost the very same question was asked for the third test, except then it was asked which object would arrive first at the bottom. The answer for that question was the sphere. Obviously, the disk arrives second.

Answer = A

2. Again, almost the very same question was asked for the third test, except then the net torque was zero. This question described the net torque as non-zero. Therefore the object must be turning with an angular acceleration. The object may or may not be accelerating linearly, depending on whether the net force was not zero too. It is possible to have a non-zero torque but zero net force. A couple always has zero net force by definition.

Answer = D

3. We are given numerical values for $x(t = 0)$, $v(t = 0)$, and $a(t) = 0$, for a simple harmonic motion for which $x(t) = A\cos(\omega t + \phi)$

$$x(t = 0) = -0.50 = A\cos\phi$$

$$v(t = 0) = -0.80 = -\omega A\sin\phi$$

$$a(t = 0) = 8.3 = -\omega^2 A\cos\phi$$

We can divide the third equation by the first equation to get ω^2

$$\omega^2 = \frac{8.3}{0.5} \quad \text{and} \quad k = m\omega^2 \Longrightarrow k = 80 \text{ N/m}$$

Answer = A

4. We use Pascal's law to compare pressures along a horizontal line through point B

$$\rho g 5 = \rho_{\text{oil}} g 20 \Longrightarrow \rho = 4\rho_{\text{oil}} = 1840 \text{ kg/m}^3$$

Answer = D

5. By inspection $k = 13.4$ m^{-1} and $\omega = 488$ rad/s. From these we can calculate $\lambda = 2\pi/k = 0.469$ m, and $T = 2\pi/\omega = 12.9$ ms.

Answer = D

6. We compute $\Delta l = (\alpha_{\text{al}} L_{\text{al}} + \alpha_{\text{st}} L_{\text{st}})\Delta T = (2.4 \times 10^{-5} * 40 + 1.2 \times 10^{-5} * 30)(95) = 1.3 \times 10^{-3}$ m

Answer = B

7. For a constant volume change we compute $Q = nC_V\Delta T = (6000)(5R/2)(430 - 270) = 2.0 \times 10^7$ J.

Answer = C

Part I Multiple Choice

8. We first compute the initial volume V_1 from the ideal gas law, and then we compute the final volume V_2 using one of the adiabatic ideal gas laws.

$$V_1 = \frac{nRT}{p_1} = \frac{(20)(8.314)(450)}{40 \times 10^3} = 0.187 \text{ m}^3$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Longrightarrow V_2 = \left(\frac{T_1}{T_2}\right)^{1/(\gamma-1)} V_1 = \left(\frac{450}{320}\right)^{3/2} (0.187) = 0.31 \text{ m}^3$$

Answer = D

9. The efficiency is the work done (enclosed area) divided by the amount of heat taken in. The positive work done during the isobaric expansion process can be computed simply as $W_1 = 9P_0 * 3V_0 = 27P_0V_0$. The negative work during the adiabatic compression is given by

$$W_2 = \frac{1}{\gamma - 1}(4P_0V_0 - 9P_0V_0) = -\frac{5P_0V_0}{0.4} = -12.5P_0V_0$$

The heat taken in during the *isobaric expansion* can be computed

$$Q_{\text{in}} = nC_p\Delta T = nC_p(T_2 - T_1) = n(7R/2)(T_2 - T_1)$$

We can figure out the two temperatures at the end and at the start of the isobaric expansion using the ideal gas law. These two temperatures are $T_2 = 36P_0V_0/nR$ and $T_1 = 9P_0V_0/nR$. With these temperatures we can compute the heat input Q_{in} as

$$Q_{\text{in}} = n\left(\frac{7R}{2}\right)\left(\frac{27P_0V_0}{nR}\right) = 94.5P_0V_0 \quad \text{(This value is independent of the number of moles.)}$$

Finally, the efficiency is given by

$$\epsilon = \frac{W_1 + W_2}{Q_{\text{in}}} = \frac{27 - 12.5}{94.5} = 0.15$$

Why is the heat transfer which takes place during the isochoric step being ignored?

Answer = D (Most difficult Multiple Choice problem with only 10% correct answers)

10. Diesel engines operate at higher compression ratios r as compared with gasoline engines. Therefore, they must be made bulkier to contain the higher pressures. The high pressure generates a high enough temperature to cause the diesel fuel to ignite without needing a spark plug.

Answer = C

11. The car and the train are approaching each other. So this Doppler equation is written as

$$f_L = \frac{v + v_L}{v - v_S} f_S = \frac{344 + 30}{344 - 50} 1.50 = 1.91 \text{ kHz}$$

Answer = C

12. When an ideal gas expands at constant pressure, then the temperature must increase. Since the average kinetic energy is directly proportional to the temperature, then an isobaric expansion of a gas means that the average kinetic energy of the gas molecules must increase.

Answer = B

Part I Multiple Choice

13. The speed of a transverse wave in a wire under tension T is given by $v = \sqrt{T/\mu}$ where μ is the mass per unit length. The speed v is also given by $v = f\lambda$. Therefore

$$\sqrt{\frac{T}{\mu}} = f\lambda \Longrightarrow T = \mu\lambda^2 f^2 = \frac{0.06}{9.0}(120)^2(0.3)^2 = 8.6 \text{ N}$$

Answer = A

14. We are given that the train is at a speed of 31 m/s when it starts to decelerate to a speed of 0 with $a = -0.065$ m/s^2. So we can figure out the distance s traveled at constant deceleration by

$$v_f^2 = 0 = v_i^2 + 2as \Longrightarrow s = \frac{-v_i^2}{2a} = \frac{(31)^2}{0.065} = 7.4 \times 10^3 \text{ m}$$

Answer = D

15. Since there is no air resistance, the horizontal component of the velocity remains constant during the entire trajectory $v_x = v_{0x} = 110$ m/s.

Answer = E

16. You push down against the Earth such that the net force on the Earth's surface is greater than your own weight. By Newton's third law, the Earth's surface pushes back on you with the same amount of force. You will have a net upward force equal to the force from the Earth's surface less your own weight. You can figure out how much force that is by knowing your mass, knowing how high you jump, and how long it took you to get to that highest point. *Can you figure out how much distance the Earth moves in the opposite direction?* (If you answer the Earth recoils the same amount of distance that you jumped up, then you have to repeat the first semester of this course.)

Answer A cannot be correct because internal forces can never cause an acceleration of a mass. Answers C and D say the same thing, but are wrong. Answer E is against common sense.

Answer = B

17. Gravity is acting because the problem states an upward acceleration. There is no up unless there is a down defined by the direction of the gravity acceleration.

$$P = M(a + g) = 50(4.0 + 9.8) = 690 \text{ N}$$

Answer = A

18. For a varying force in one dimension work is calculated as

$$W = \int_{x_1}^{x_2} F(x)\,dx = \int_0^{0.27} 8x^3\,dx = 2x^4|_0^{0.27} = 1.06 \times 10^{-2} \text{ J}$$

Answer = A

Part I Multiple Choice

19. Simple conservation of energy $m_1gh = m_1v^2/2$ and $m_2gh = m_2v^2/2$ means that having $m_2 = 2m_1$ will give m_2 twice the kinetic energy for a fall from the same height h.

Answer = A

20. A completely inelastic (or any inelastic) collision conserves linear momentum but not kinetic energy.

Answer = C

21. The Coke can imploded because a partial vacuum was created when the steam vapor inside the can suddenly condensed.

Answer = D (Easiest Multiple Choice problem with 95% correct answers)

Common Errors Made in the Part II Problem Solutions

Complete Part II solutions are on the following 4 pages

Problem 1

a) The isothermal portion must be a curved (hyperbola shape) line, not a straight line.
b) The isothermal work formula was not used. The isobaric work formula was not used. The MKS version of the gas constant R was not used, but answers were quoted in Joules when they were really being calculated as liter-atm because the "chemistry" version of R was used.
c) The (positive) heats of isothermal work and isochoric change were not calculated.
d) The (negative) heat of an isobaric compression was not calculated.
e) Use of Carnot efficiency formula for a non-Carnot engine.

Problem 2

c) There is no tangential acceleration $a_t = r\alpha$ because there is no angular acceleration α. The angular velocity ω is constant in this problem.

Problem 3

a) The effect of the vertical component of the pull force P was neglected when calculating the normal N force of the surface on the block. Therefore the friction force was mis-calculated and this mistake caused the acceleration to be mis-calculated.

Problem 4

b) The flow rate formula Av was not used. The diameter (6.35 cm) was used instead of the radius in $A = \pi r^2$.
c) No calculations were needed. The pressure in the external region is always air pressure.
d) Bernoulli's equation was not used at all, or the correct values from parts a), b), and c) were not used in Bernoulli's equation. Some answers were wildly overestimated. An ordinary firehose is not able to contain more than a few atmospheres overpressure of water.

Final grades should be in the Oak system by no later than Friday.

Part II Worked Problems. 25 points each

1. One mole of an ideal, mono–atomic gas is taken through a pV cycle $A \rightarrow B \rightarrow C \rightarrow A$. Process $A \rightarrow B$ is an **isothermal** expansion, process $B \rightarrow C$ is an **isobaric** compression at 1.0 atm ($= 1.013 \times 10^5$ N/m^2) from 50 liters to 10 liters (1 liter $= 10^{-3}$ m^3), and process $C \rightarrow A$ is an **isochoric** pressure change from 1.0 to 5.0 atm at a volume of 10 liters. Each part is worth 5 points.

a) Draw the complete pV diagram, after you compute the three pairs of (p, V) values at A, B, and C based on the information provided and using the ideal gas law.

From the problem statement we are given the following (p, V) information: $p_A = 5.0$ atm, $V_A = 10$ l; $p_B = 1.0$ atm, $V_B = 50$ l; $p_C = 1.0$ atm, $V_C = 10$ l. Furthermore, we are told that the $A \rightarrow B$ process is isothermal. We could have figured out that particular fact independently by seeing that $p_A V_A = p_B V_B$. So it a simple matter to construct the *closed cycle* pV diagram, using these three pairs of data points. The $A \rightarrow B$ curve is a hyperbola, the $B \rightarrow C$ path is a horizontal straight line at $p_B = p_C = 1.0$ atm, and the $C \rightarrow A$ path is a vertical straight line at $V_C = V_A = 10$ l.

We can calculate the temperatures at each point from the ideal gas law, using MKS units

$$T_A = T_B = \frac{p_A V_A}{nR} = \frac{(5.0 \times 1.013 \times 10^5)(10 \times 10^{-3})}{(1)(8.3145)} = 610 \text{ K}$$

Of course you should get the same answer using the liter–atmosphere units

$$T_A = T_B = \frac{p_A V_A}{nR} = \frac{(5.0)(10)}{(1)(0.082)} = 610 \text{ K} \quad \text{and} \quad T_C = \frac{p_C V_C}{nR} = \frac{(1.0)(10)}{(1)(0.082)} = 122 \text{ K}$$

b) What is the *net work* done by the gas ?

You have to realize that (positive) work is done by the gas during the isothermal expansion process $A \rightarrow B$, while (negative) work is done on the gas during the isobaric compression process $B \rightarrow C$. No work is done during the isochoric process $C \rightarrow A$.

$$W_{AB} = nRT_A \ln\left(\frac{V_B}{V_A}\right) = (1)(8.3145)(610)\ln(5) = 8,163 \text{ Joules}$$

$W_{BC} = p_B(V_B - V_C) = (1.0 \times 1.013 \times 10^5)(-40 \times 10^{-3}) = -4,052 \text{ J} \Longrightarrow W_{net} = W_{AB} + W_{BC} = +4,111 \text{ J}$

c) How much heat is *added* to the gas ?

Heat is added to the gas during the isothermal expansion, and all of that heat goes into doing work: $Q_{AB} = W_{AB} = +8{,}160$ Joules. Heat is also added to the gas during the isochoric step Q_{CA} which raises the temperature from 122 K to 610 K: $Q_{CA} = nC_V\Delta T = (1)(3R/2)(T_A - T_C) = (1)(1.5 * 8.3145)(+488) = +6,086$ Joules. So the total amount of heat added to the gas is 14,246 Joules.

d) How much heat is *expelled* by the gas ?

Heat is expelled from the gas during isobaric compression,during which the temperature drops: $Q_{BC} = nC_p\Delta T = (1)(5R/2)(T_B - T_C) = (1)(2.5 * 8.3145)(-488) = -10,144$ Joules. Check that $Q_{AB} + Q_{BC} + Q_{CA} = 8,163 - 10,144 + 6,086 = +4,105$ Joules, which is the net work done (within round-off errors).

e) What is the efficiency of this "heat engine" ?

$$\text{Efficiency} \quad \epsilon \equiv \frac{W_{net}}{Q_{in}} = \frac{4,105}{14,246} = 29\%$$

2. An 8.0 cm (radius) disk rotates at a constant rate of 1200 revolutions/minute about its axis. Part a) is worth 7 points, parts b–d worth 6 points

a) What is the angular velocity of the disk?

You could simply say that $\omega = 1200$ revolution/minutes, or change to the usual radians/second since that will be needed for future calculations:

$$\omega = \frac{1200 \times 2\pi}{60} = 126 \text{ radians/second}$$

b) What is the linear speed of a point 3 cm from the center of the disk?

$$v = r\omega = (3)(126) = 377 \text{ cm/s}$$

c) What are the radial and tangential accelerations of a point on the rim of the disk?

$$a_r = \frac{v^2}{r} = r\omega^2 = (8)(126)^2 = 1.26 \times 10^5 \text{ cm/s}$$

$$a_t = r\alpha = 0$$

In this problem with **constant** angular velocity, then there is no angular acceleration and no tangential acceleration either.

d) What is the total distance that a point on the rim moves in 2 seconds?

You can quote the distance in either radians, or cm, since the question did not specify angular or linear distance.

$$\text{Angular distance} \;\; \theta(t) - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2 = (126)(2) = 251 \text{ radians}$$

The linear distance would be the radius times this angular distance, which gives 2.01×10^3 cm

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3. A 40 kg mass initially at rest is pulled a distance of 5 meters along a rough ($\mu_k = 0.10$) horizontal floor with a constant force of 130 N at an angle of 15^{O} above the horizontal. Each part below is worth 5 points.

a) What is the acceleration, if any, of the mass ?

For a net horizontal force F acting on a mass m, Newton's second law states

$$F = ma \Longrightarrow a = \frac{F}{m}$$

The net horizontal force acting is the horizontal component of the pull force P less the opposing kinetic friction force f_k The horizontal component for P is

$$P \cos\theta = 130 \cos(15) = 126 \text{ N}$$

The frictional force is the coefficient of kinetic friction times the normal force of the table N. That normal force N is the weight of the mass less the vertical component of P

$$f_k = \mu_k N = \mu_k(mg - P\sin\theta) = 0.10(40 * 9.8 - 130\sin(15)) = 35.8 \text{ N}$$

$$a = \frac{126 - 35.8}{40} = 2.2 \text{ m/s}^2$$

b) What is the total work done by the applied force ?

The applied force P does work

$$W_P = Pd\cos\theta = (130)(5)\cos(15) = 628 \text{ J}$$

c) What is the (negative) work done by the frictional force ?

The frictional force does (negative) work

$$W_f = f_k d\cos(180) = -179 \text{ J}$$

d) What is the kinetic energy of the mass at the end of the 5 meter distance ?

The kinetic energy is the difference between the last two results

$$K = W_P - W_f = 449 \text{ J}$$

e) What is the final speed of the mass ?

The speed of the mass can be found directly from the kinetic energy

$$K = \frac{1}{2}mv_f^2 \Longrightarrow v_f = \sqrt{\frac{2K}{m}} = 4.7 \text{ m/s}$$

The speed of the mass can also be found from the acceleration

$$v_f^2 = v_0^2 + 2ad \Longrightarrow v_f = \sqrt{2(2.2)5} = 4.7 \text{ m/s}$$

4. To fight a fire on the fourth floor of a building, a firefighter wants to use a hose of diameter 6.35 cm (2.5 inches) to shoot 950 liters/minute to a height of 12 meters. Part a is worth 7 points, parts b–d worth 6 points

(1 liter $= 10^{+3}$ cm^3 $= 10^{-3}$ m^3, $\rho_w = 1000$ kg/m^3).

a) What is the minimum speed with which the water must leave the nozzle of the fire hose (pointed straight up) if the water is to go up 12 meters ?

For any mass m given an initial vertical speed v, the mass m will reach a height h according to conservation of energy

$$mgh = \frac{1}{2}mv^2 \Longrightarrow v = \sqrt{2gh} = \sqrt{2(9.8)12} = 15.3 \text{ m/s}$$

We will call this speed v_2

b) According to the given flow (950 l/min) rate, what is the speed of the water inside the hose ?

The flow rate is given by Av_1, which in m^3/s is calculated as

$$\text{Flow Rate} \quad = \frac{950 \times 10^{-3}}{60} = Av_1 = \pi r^2 v_1 = \pi((6.35 \times 10^{-2}/2)^2)v_1 \Longrightarrow$$

$$v_1 = (\frac{950 \times 10^{-3}}{60})/(\pi * (6.35 \times 10^{-2})^2/4.0) = 5.0 \text{ m/s}$$

c) What **must** be the pressure of the water just outside the hose (*i.e.* in the air) ?

In the open air, the water pressure must be the same as the air pressure, namely 1 atmosphere, or 1.013×10^5 Pa.

Of course, if you put your hand in front of a firehose, then there would be a lot of force on your hand pushing it out of the way. This force would be coming from the water as your hand tries to stop or reflect the water flow backwards, meaning change the momentum of the water. You would not be able to exert enough force with your hand to do that. Your hand would be pushed aside by the flow of the water.

d) What is the pressure of the water inside the hose ?

We calculate the pressure of the water just inside the hose using Bernoulli's law, assuming that there is no change in the vertical height of the water

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

From above, $v_1 = 5.0$ m/s, $P_2 = 1.013 \times 10^5$ Pa, and $v_2 = 15.3$ m/s.

$$P_1 = 1.013 \times 10^5 + 500(235.2 - 25) = 2.06 \times 10^5 \text{ Pa}$$

which is 2.04 atmospheres total, or just over 1 atmosphere above normal air pressure.

Can you understand why the nozzle on a firehose is constructed the way it is, with a fairly long tapered shape?