REVIEW: (Chapter 12) Newton's Law of Gravity

Newton deduced his Law of Universal Gravity based on the astronomical observations made in the previous two centuries.

Law of Universal Gravitation

$$F_G = G\frac{m_1 m_2}{r^2} \qquad G = 6.674 \pm 0.001 \times 10^{-11}\text{N–m}^2/\text{kg}^2$$

This force has infinite range, and also an *inverse square* dependence.

Gravity acceleration as a function of altitude

The g constant of gravitational acceleration has a value of 9.8 m/s^2 on the Earth's surface. However, g will get smaller with altitude. Suppose we are at a distance h above the Earth's surface. What will the value of g be then?

We first solve for g at the Earth's surface. In that case the r in Newton's Law of Universal Gravitation is R_E. We take the weight force of a mass m. On the Earth's surface we have

$$w(r = R_E) = mg(r = R_E) = G\frac{mM_E}{R_E^2}$$

$$g(r = R_E) = G\frac{M_E}{R_E^2}$$

Now consider an altitude h such that $r = R_E + h$. We now have

$$w(r = R_E + h) = mg(r = R_E + h) = G\frac{mM_E}{(R_E + h)^2}$$

$$g(r = R_E + h) = G\frac{M_E}{(R_E + h)^2} = g\frac{R_E^2}{(R_E + h)^2}$$

So if we put in a value $h = 2R_E$, an altitude of twice the Earth's radius we get

$$g(r = 3R_E) = g\frac{R_E^2}{(3R_E)^2} = \frac{g}{9}$$

So an object weighing 270 N on the Earth's surface will weigh only 30 N when placed at an altitude $h = 2R_E$ twice the Earth's radius.

Examples of gravity acceleration on other bodies

Gravity on the Moon, Mars, and Jupiter

The gravitational acceleration g_m at the surface of some spherical mass m depends on the radius of that mass and the size of that mass. For the Earth's surface we have

$$g_E = G\frac{M_E}{R_E^2} = 6.67 \times 10^{-11}\frac{5.97 \times 10^{24}}{(6.38 \times 10^6)^2} = 9.8 \text{ m/s}^2$$

For the Moon's surface we have

$$g_M = G\frac{M_M}{R_M^2} = 6.67 \times 10^{-11}\frac{7.35 \times 10^{22}}{(1.74 \times 10^6)^2} = 1.6 \text{ m/s}^2$$

The Moon's surface gravitational acceleration is about 1/6 of the Earth's gravitational acceleration For Jupiter's surface we have

$$g_J = G\frac{M_J}{R_J^2} = 6.67 \times 10^{-11}\frac{1.90 \times 10^{27}}{(6.91 \times 10^7)^2} = 26.5 \text{ m/s}^2$$

Jupiter's surface gravity is "only" three times that of the Earth's, even though Jupiter is 300 times more massive than the Earth. The relatively small increase in g given the large mass increase is because of the much bigger radius of Jupiter. Now, consider a human being of mass 70 kg rolled up into more or less a spherical shape with a radius of 0.30 m. A human gravitational acceleration is

$$g_H = G\frac{M_H}{R_H^2} = 6.67 \times 10^{-11}\frac{70}{(0.3)^2} = 5.2 \times 10^{-8} \text{ m/s}^2$$

So two humans attract each other with a force of roughly 8 orders of magnitude below the force that the Earth attracts one of them.

Gravity and Astrology

Consider a star say 4.35 light-years away. The system Alpha-Centauri, which are actually 3 stars, is at that distance which are the closest stars to our solar system. What is the gravitational acceleration on us from one of those stars, which has the same mass as our Sun?

$$g_{A-C} = G\frac{M_{A-C}}{r^2} = 6.67\times10^{-11}\frac{1.99 \times 10^{30}}{(4.35 * 3 \times 10^8 * 365 * 24 * 3600)^2} = 4.5\times10^{-11} \text{ m/s}^2$$

The gravitational acceleration from the Earth's nearest star is three orders of magnitude less than the gravitational acceleration of your nearest neighbor. *"The fault, dear Brutus, is not in our stars but in ourselves..."*
[Wm. Shakespeare in Julius Caesar I,ii,140–141, 70 years before Isaac Newton]

Universal Gravity from Real Spherical Masses

Solar System Dimensions

In our solar system the almost spherical Sun has a mass of $M_S = 1.99 \times 10^{30}$ kg, and a radius of 6.96×10^8 m. For comparison the Earth–Moon radial distance is 3.84×10^8 m, just more than half the size of the Sun's radius alone.
The Earth's orbital radius is 1.50×10^{11} m, more than 200 times the Sun's radial size. So you might think that it is only a good approximation, at the half-percent level, to think of the Sun as a point mass in Newton's Universal Gravity Law as a function of distance r away from the center of the Sun

$$F_G(r) = Gm_pM_S/r^2$$

In fact even if the Sun's radius were almost the orbital radius of the Earth, it is exactly correct to regard the Sun's mass as all at its center for purposes of calculating Newton's Law of Gravity on the Earth!

Results from Calculus for Newton's Universal Law of Gravity

As you know, Newton's invented calculus to help him in proving the mathematical consequences of his Universal Law of Gravity. Using calculus Newton first proved two rules for dealing with the gravitational force from a spherical mass M which has a spherical radius R_M. These two rules are:

1) For radial distances $r \geq R_M$ of a mass m from M, the force of gravity can be computed as if all the mass M was at the center of the sphere

$$\text{Force outside sphere's radius: } F_G(r \geq R_M) = G\frac{mM}{r}$$

2) For radial distances $r \leq R_M$ of a mass m from M, meaning m is *inside* of M, then the force of gravity is computed by taking that part of the mass of M which is located inside of radial distance r

$$\text{Force inside sphere's radius: } F_G(r \leq R_M) = G\frac{mM(r)}{r}$$

If the mass M has a uniform spherical density ρ_M, we can calculate

$$\rho_M = \frac{M}{V} = \frac{M}{4/3\pi R^3} \Longrightarrow M(r) = \rho V(r) = \frac{M}{4/3\pi R^3} \times \frac{4}{3}\pi r^3 = \frac{r^3}{R^3}M$$

$$\Longrightarrow \text{Force inside sphere's radius: } F_G(r \leq R_M) = G\frac{m}{r^2}(\frac{r^3}{R^3}M) = G\frac{mMr}{R^3}$$

The gravity force at the center of a sphere (*i.e.* $r = 0$) is zero!

Energy in Planetary Orbits

Kinetic Energy for a Planet in Circular Orbit due to the Sun's Gravity

All planets have **negative** *mechanical energy?*

How can this be? It surely can't be because of the Kinetic Energy $K = \frac{1}{2}m_p v^2$ which has to be positive because all planets have a non-zero speed. We can work out the formula for the speed of a planet in circular orbit of radius R, as we did for the Kepler's Third Law derivation

$$F_{\text{centripetal}} = \frac{m_p v^2}{R} = F_{\text{Gravity}} = G\frac{m_p M_S}{R^2}$$

$$\Longrightarrow v^2 = G\frac{M_S}{R} \Longrightarrow v = \sqrt{G\frac{M_S}{R}}$$

You can see in the above equation that the speed of a planet in orbit about the Sun is independent of the mass of the planet. At a given radius R from the Sun, all objects in circular orbit will have the same speed. So in the asteroid belt between Mars and Jupiter, all the asteroids have about the same speed if they are at the same distance from the Sun in circular orbit.

Potential Energy for a Planet in Circular Orbit due to Sun's Gravity

If the total energy is negative then it must the the "fault" of the potential energy. For the Universal Gravity force, we can derive from the integral definition that the potential energy is given by the formula

$$U \equiv -\int F(r)dr \Longrightarrow U_G^S(R) = -G\frac{m_p M_S}{R}$$

The absolute value, or zero reference, of potential energy is arbitrary. Normally we say the gravity potential energy is zero when two masses are infinitely far apart. In the above formula, m_p is the mass of the planet, R is the orbital radius of the planet (distance from the Sun), and M_S is the mass of the Sun.

Total Energy for a Planet in Circular Orbit due to the Sun's Gravity

We can get the total energy for a planet in circular orbit about the Sun by summing up the kinetic and the potential energies

$$E = \frac{1}{2}m_p v^2 + U_G^S(R) = \frac{1}{2}m_p G\frac{M_S}{R} - G\frac{m_p M_S}{R} = -\frac{1}{2}G\frac{m_p M_S}{R}$$

The total energy of a planet in circular orbit about the Sun is negative, and equal to one have the potential energy. Equivalently, the total energy of a planet in circular orbit about the Sun is equal to the negative of the kinetic energy.

Energy in Planetary Orbits

The Escape Speed from the Earth's Surface

We can compute the total energy of a particle of mass m at the Earth's surface by similarly summing up the kinetic energy and the potential energy due to the Earth's gravity. For that potential energy we use the previous potential energy formula, but substituting the Earth's mass M_E for the Sun's mass, and using the Earth's radius R_E for the R value.

$$U^E_G(R_E) = -G\frac{mM_E}{R_E}$$

For a given initial speed v_i at the Earth's surface where $r_i = R_E$ we have the total energy conserved at any later distance r:

$$E = E_i = \frac{1}{2}mv_i^2 - G\frac{mM_E}{R_E} = \frac{1}{2}mv^2 - G\frac{mM_E}{r} = E(\text{ arbitrary distance } r)$$

To get the maximum distance r_{max} we set the "final" speed equal to 0. This then gives us the expression for r_{max}

$$\frac{1}{r_{max}} = \frac{1}{GmM_E}(\frac{1}{2}mv_i^2 - G\frac{mM_E}{R_E})$$

Now suppose we want the mass to travel infinitely far away for the Earth, so then $1/r_{max} = 0$. Then the expression above in parenthesis must also be zero:

$$(\frac{1}{2}mv_i^2 - G\frac{mM_E}{R_E}) = 0 \Longrightarrow v_i(r_{max} = \infty) \equiv v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

Objects with a speed v_{esc} at the Earth's surface will travel to infinity and never return. Obviously, you can substitute for any other planet or even the Sun to get their escape speed. The concept of escape speed is critical to understand why the moon has no atmosphere, why the Earth has no hydrogen or helium in its atmosphere, and why the outer heavy planets all contain hydrogen and helium as their most abundant elements.

For the Earth $v_{esc} = 1.12 \times 10^4$ m/s, which is about 6.9 miles/second. The magnitude of the escape velocity is crucial for a planet to retain an atmosphere. The escape velocity for Mars is about half that of the Earth, and Mars has much less of an atmosphere compared to Earth.

The Ultimate in Escape Speed: The BLACK HOLE

According to relativity, the fastest speed anything can have is the speed of light c. Nothing, not even light, can go faster then c. So what happens when the escape speed becomes bigger than the speed of light? What you have then is something called a **Black Hole** ! This is an object so massive, and so compact, that nothing can escape from it not even light itself. So you can't see it, but it's there. And if you get to close to it, then you will never escape yourself.

Do **Black Holes** exists? Most physicists think so. It is believe that Stars with a mass several times that of the Sun will eventually explode into a "Super-nova" and leave behind an incredibly dense core. For example, the core might be the *mass of the Earth and the size of a dime.* The only evidence for a **Black Hole** would be if another object is too close, like a binary star companion, and one sees the companion gradually consumed by the **Black Hole**.

Artificial Satellites in Earth Orbit

Gravity Force at Radial Distance r From Earth's Center

An artificial satellite can be in orbit at a distance r from the Earth's center. Typically we quote the altitude h above the Earth's surface which is R_E away from the Earth's center.

The gravity force at altitude h is then

$$F_G(R_E + h) = G\frac{m_s M_E}{(R_E + h)^2}$$

where $R_E = 6.38 \times 10^6$ m, $M_E = 5.97 \times 10^{24}$ kg, and m_s is the mass of the satellite.

For a satellite in circular orbit, the required centripetal force is

$$F_{\text{centripetal}} = \frac{m_s v^2}{r} = \frac{m_s v^2}{R_E + h}$$

$$F_{\text{centripetal}} = F_G(R_E + h) = G\frac{m_s M_E}{(R_E + h)^2} = \frac{m_s v^2}{R_E + h}$$

Speed and Orbital Period of an Earth Satellite

The speed of an Earth satallite at altitute h can be now obtained as

$$v(h) = \sqrt{\frac{GM_E}{R_E + h}}$$

This speed depends only on the altitude h, and not on the mass of the satellite. So when an astronaut leaves the Space Shuttle for a "space walk" the astronaut is still traveling in orbit with the same speed as the space shuttle. For a satellite or space shuttle at altitude 300 km in circular orbit, the speed is

$$v(h = 3 \times 10^5 \text{ m}) = 7.72 \times 10^3 \text{ m/s}$$

The oribtal time T is the circumfrence $2\pi r$ divided by the speed

$$T(h) = \frac{2\pi r}{v} = \frac{2\pi(R_E + h)}{v(h)} = 2\pi(R_E + h)\sqrt{\frac{R_E + h}{GM_E}}$$

Again, with $h = 300$ meters, we can calculate the orbital time $T(300 \text{ m}) = 5440$ s, or about 91 minutes. Satellites in these so-called "near-Earth" orbits typically take one and one-half hours per orbit. On the other hand, satellites much further away, about 22,500 miles, are in *synchronous* orbits, meaning $T =$ 24hours. Can calculate the altitude for a synchronous orbit?

Kepler's Three Laws

Before Newton formulated his law of universal gravitation, the astronomer Kepler had worked out three laws of planetary motion. As far as Kepler was concerned, these were three independent laws. However, Newton showed that all three of Kepler's Laws could be derived from Newton's one law of gravity. So Newton accomplished the first "unification" in physics.

Kepler's First Law

Each planet moves in an elliptical orbit with the Sun at one focus

Actually, all the planets exceptformer planet Pluto, have very near circular orbits. The elliptical orbit can be shown to be the general solution to Newton's gravity law with an inverse square dependence.

Kepler's Second Law

The radius vector from the Sun to a planet sweeps out equal areas in equal amounts of time.

This is know as the "equal areas" law. In fact, it is simply a consequence of the fact that the gravity force is a central force. It exerts no torque on the planet. Then it can be shown that the areas swept out are just proportional to the angular momentum which is conserved. The equal areas law does not depend on the inverse square dependence.

Kepler's Third Law

The square of the period T of a planet's orbit is proportional to the cube of its semi-major axis.

This was originally thought to be some magical property of the planets ("harmony of the spheres" was the phrase used in Kepler's time) but actually it's an extremely simple consequence of the inverse square force of gravity law.

Kepler's Third Law Proved

It is very easy to prove Kepler's Third Law for circular orbits
The square of the period T of a planet's orbit is proportional to the cube of its radius R.
We take the mass of the Sun to be M_S and the mass of the planet to be m_p. The orbital radius is taken as R. Newton's gravity law states

$$F = G\frac{M_S m_p}{R^2}$$

However, this gravity force is also the centripetal force, just as we saw for the moon. And centripetal forces have simple expressions in terms of the speeds:

$$F = \frac{m_p v^2}{R}$$

Now the speed v is just the circumference of the orbit divided by the time of the orbit

$$v = \frac{2\pi R}{T}$$

So equate everything concerning the force:

$$F = G\frac{M_S m_p}{R^2} = \frac{m_p v^2}{R} = \frac{m_p 4\pi^2 R^2}{T^2}$$

The m_p term cancels out and we are left with

$$G\frac{M_S}{R^2} = \frac{4\pi^2 R^2}{T^2}$$

$$T^2 = \frac{4\pi^2 R^3}{G M_S} = K_S R^3$$

$$K_S \equiv \frac{4\pi^2}{G M_S}$$

For planetary orbits the cube of the Radius
Is proportional to the square of the Period

The square of the time is proportional to the cube of the radius. Moreover, once you have found out what G value is, then you can determine from K_S what the mass of the Sun is.

CHAPTER 13: Oscillatory Motion

Consider a spring lying in a horizontal position. A mass is attached to the spring, and the spring is stretched and then released. In the absence of friction the mass will *oscillate* along the horizontal axis. That means the position x of the mass will go from a maximum positive displacement, to zero displacement, and then to a maximum negative displacement, and then repeat the *cycle*. This position function $x(t)$ obeys the *Simple Harmonic Motion* equation (SHM):

$$\textbf{Simple Harmonic Motion} \quad x(t) = A\cos(\omega t + \phi) \tag{13.13}$$

The position function $x(t) = A\cos(\omega t + \phi)$ has three parameters:

$A \equiv$ is the **amplitude** of the motion

$\omega(= 2\pi f) \equiv$ is the **angular frequency** of the motion

$\phi \equiv$ is the **phase constant** of the motion

The time for one complete *cycle* of the oscillation is called the **period T**.
The number of cycles per second is called the **frequency f**. The frequency f is the inverse of the period T

$$\textbf{frequency} \quad f = 1/T$$

There are two simple systems, *pendulum* and *spring* for which you should now the equation for their periods in terms of the physical parameters:

Period for a Spring of a given mass and force constant: $T = 2\pi\sqrt{\dfrac{m}{k}}$

$$\textbf{Period for a Pendulum} \text{ of a given length: } T = 2\pi\sqrt{\frac{L}{g}} \tag{13.34}$$

The velocity and the acceleration of the mass in oscillatory motion can be calculated directly from the position function $x(t) = A\cos(\omega t + \phi)$ by taking the first and the second time derivatives respectively:

$$v(t) = \frac{dx}{dt} = \frac{d(A\cos(\omega t + \phi))}{dt} = -\omega A\sin(\omega t + \phi) \tag{13.15}$$

$$a(t) = \frac{dv}{dt} = \frac{d(-\omega A\sin(\omega t + \phi))}{dt} = -\omega^2 A\cos(\omega t + \phi) \tag{13.16}$$

Oscillatory or Simple Harmonic Motion

The general equation for simple harmonic motion is given by

$$x(t) \quad (\text{or } y(t) \ ) = A\cos(\omega t + \phi) \tag{13.13}$$

This equation describes the motion of a mass attached to a spring (either horizontally or vertically), or the motion of a pendulum.
The purpose of this chapter is to study this type of motion, and in particular to become familiar with the concepts of *Amplitude*, *frequency*, and *phase*.

Take a specific case of a spring which is stretched to an initial distance x_0, and an attached mass is also given an initial speed v_0. We will see that the constants A and ϕ can be expressed in terms of the two initial conditions x_0 and v_0.
We first substitute in the position and the velocity equations at time $t = 0$

$$x(t=0) = A\cos(\omega \cdot 0 + \phi) = A\cos\phi = x_0$$

$$v(t=0) = -\omega A\sin(\omega \cdot 0 + \phi) = -\omega A\sin\phi = v_0$$

Now divide the second equation by the first in order to get an equation for the phase angle by itself in terms of x_0 and v_0

$$\tan\phi = -\frac{v_0}{\omega x_0}$$

With a little more work, you can substitute this expression for $\tan\phi$ into one of the two other equations and obtain an expression for the amplitude A just in terms of x_0, v_0, and ω

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$$

Special case where $v_0 = 0$ (no initial velocity)

$$\tan\phi = 0 \Longrightarrow \phi = 0$$

$$A = \sqrt{x_0^2 + 0} \Longrightarrow A = x_0$$

So this special case gives a very simple Simple Harmonic Motion equation

$$x(t) = x_0\cos\omega t \qquad\qquad (\text{only when } v_0 = 0)$$

Worked example of Simple Harmonic Motion

A particle oscillates in simple harmonic motion according to the following position equation where t is in seconds and x is in meters:

$$x(t) = 4.0\cos{(\pi t + \frac{\pi}{4})}$$

Here the *phase angle* ϕ is given in radians instead of the more common degrees. (Recall that π radians is equal to $180^{\rm O}$, so $\pi/4 = 45^{\rm O}$.)
Determine the amplitude, frequency (and angular frequency), and period of the motion. Determine the position, velocity, and acceleration of the particle at $t = 1$ second.

The amplitude A and the angular frequency ω can be determined by simply comparing this equation to the general equation $x(t) = A\cos{(\omega t + \phi)}$. You will see right away that in this example $A = 4.0$ meters, and $\omega = \pi$ radians/second. To get the (plain) frequency f, and then the period T requires that you recall the relation between the angular frequency ω and f

$$\omega = 2\pi f \Longrightarrow f = \frac{\omega}{2\pi} \tag{13.11}$$

So in this case $f = \pi/(2\pi) = 0.5$ cycles per second.
The period is given just as the inverse of the (plain) frequency f

$$T = \frac{1}{f} = \frac{1}{0.5\ \text{s}^{-1}} = 2.0\ \text{s}$$

To get the position and velocity at $t = 1$ second, just substitute in position and the velocity equations of motion:

$$x(t=1) = 4.0\cos{(\pi\cdot 1 + \frac{\pi}{4})} = -2.83\ \text{m}$$

$$v(t=1) = -(4.0)(\pi)(\sin{(\pi\cdot 1 + \frac{\pi}{4})}) = 8.89\ \text{m/s}$$

$$a(t=1) = -(4.0)(\pi)^2(\cos{(\pi\cdot 1 + \frac{\pi}{4})}) = 27.9\ \text{m/s}^2$$

Newton's Second Law and Simple Harmonic Motion

Up to now we have been working with the position equation

$$x(t) = A\cos(\omega t + \phi) \tag{13.13}$$

for oscillatory motion, and have simply stated that this is the correct equation. Now we will prove that is the correct for the case of a mass attached to a spring. This we will do using Newton's second law of motion, the force equation
When a spring is stretched a distance x where $x > 0$, then there will be a force in the negative x direction. Such a force is called a **Restoring Force** because it tends to restore the spring back to its original configuration. We have already seen in Chapter 7 that the magnitude of the force depends upon the spring constant k

$$F_{spring} = -kx \tag{13.3}$$

This force is exerted on the attached mass m, and by Newton's second law we have

$$F = ma = -kx \Longrightarrow a = -\frac{k}{m}x \tag{13.4}$$

Like the force itself, the acceleration is linearly proportional and opposite in sign to the displacement from the equilibrium position.
This equation can be re–written using the calculus definition of acceleration

$$a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

We now symbolize the ratio k/m by the symbol ω^2, and then obtain:

$$\omega \equiv \sqrt{\frac{k}{m}} \Longrightarrow \frac{d^2x}{dt^2} = -\omega^2 x$$

Newton's Second Law and Simple Harmonic Motion

We have derived an equation for the position function $x(t)$:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

This equation states that we need a function $x(t)$ such that when we take two derivatives with respect to t, then we will get back the *negative* of the original function $x(t)$ multiplied by ω^2. Very simply, Eq. 13.3 is exactly that function, and the prove is just do it.
Start with Eq. 13.13, and then take two time derivatives:

$$x(t) = A\cos(\omega t + \phi) \tag{13.13}$$

$$\frac{d}{dt}x(t) = -A\omega\sin(\omega t + \phi)$$

$$\frac{d^2}{dt}x(t) = -A\omega^2\cos(\omega t + \phi) = -\omega^2 x(t)$$

In general, whenever one has a restoring force equation coming in Newton's second law, then one will always find the solution to be the position function is just simple harmonic motion.
Note that the constants A and ϕ are completely arbitrary. Any pair on numbers A and ϕ will satisfy the restoring force equation. In order to fix A and ϕ to particular values, on must specify the initial conditions, namely the initial position (x_0), and the initial velocity (v_0) as we have done previously.

The Mass–Spring System in Simple Harmonic Motion

The general solution for simple harmonic motion is

$$x(t) = A\cos(\omega t + \phi) \tag{13.13}$$

where for the spring we remember that

$$\omega^2 = \frac{k}{m} \Longrightarrow \omega = \sqrt{\frac{k}{m}} \tag{13.9}$$

Now in general $\omega = 2\pi f$ so

$$\omega = 2\pi f = \sqrt{\frac{k}{m}} \Longrightarrow f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{13.11}$$

Finally one can determine the period T of the motion in terms of k and m since $T = 1/f$

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}} \tag{13.12}$$

Worked Example A car of mass 1300 kg has four shock absorber springs each with a force constant of 20,000 N/m. If two people riding in the car have a combined mass of 160 kg, what is the frequency f of the vibration of the car when it is driven over a pothole?
Assume that the total mass (=1460 kg) is equally distributed over the four shocks, so each shock absorber has a mass of 325 kg attached to it. The solution is just to use Eq. 13.11 to solve for f

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{20,000}{365}} = 1.18 \text{ Hz}$$

The abbreviation Hz (after Heinrich Hertz) means 1 cycle/second. The period of the vibration T is simply given as the inverse of the frequency

$$T = \frac{1}{f} = \frac{1}{1.18} = 1.70 \text{ seconds}$$