CHAPTER 1: Units, Physical Quantities, Dimensions

1. PHYSICS is a science of measurement. The things which are measured are called **physical quantities** which are defined by the describing how they are to be measured. There are three fundamental quantities in **mechanics**:

Length
Mass
Time

All other physical quantities combinations of these three basic quantities.

2. All physical quantities **MUST** have units attached to them. The standard system of units is called the **SI** (Systeme Internationale), or equivalently, the **METRIC** system. This system uses

Length in Meters (m)
Mass in Kilograms (kg)
Time in Seconds (s)

With these abbreviations for the fundamental quantities, one can also be said to be using the MKS system.

3. An example of a derived physical quantity is **Density** which is the mass per unit volume:

$$\text{Density} \equiv \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass}}{(\text{Length})\cdot(\text{Length})\cdot(\text{Length})}$$

4. Physics uses a lot of formulas and equation. A very powerful tool in working out physics problems with these formulas and equations is **Dimensional Analysis**. The left side of a formula or equation **must** have the same dimensions as the right side in terms of the fundamental quantities of mass, length and time.

5. A very important skill to acquire is the art of *guesstimation*, approximating the answer to a problem. Related to that is an appreciation of sizes. Is the answer to a problem *orders of magnitude* too big or too small.

The Standards of Length, Mass, and Time

The three fundamental physical quantities are length, mass, and time.

MASS

The standard mass of 1 Kilogram (kg) is defined as the mass of a platinum–iridium alloy cylinder (3.9 cm diameter, 3.9 cm height) kept at the International Bureau of Weights and Measures at Sevres, France.

All countries have duplicates, or *secondary standards* kept at their own domestic bureaus of standards. Finally, there are *tertiary standards* which are available in all scientific laboratories.

TIME

The standard unit of time, 1 Second (s), used to be defined in terms of the time it took for the earth to rotate about its axis. Since the earth's rotation is now known to be slowing down, that is hardly a good standard. Instead the standard second is defined in terms of the **vibrations of the cesium–133 atom**. Specifically

$$1 \text{ Second} \equiv 9{,}192{,}631{,}770 \text{ vibrations}$$

In fact this a very a useful definition since any laboratory can set up a cesium clock and calibrate its time measuring equipment.

LENGTH

Formerly, like the mass definition, the definition of the unit length used to be in terms of a platinum–iridium bar kept in France. Later that was changed in terms of the *wavelength* of the orange–red light emitted from a krypton–86 lamp. Most recently, the unit of length, the meter (m), has been defined in terms of the distance traveled by light:

$$1 \text{ Meter} \equiv \text{Distance traveled by light in vacuum during } \frac{1}{299{,}792{,}458} \text{ seconds}$$

In principle, all the units except mass, can defined worldwide without reference to any particular object.

The abbreviations of the fundamental quantities of length, mass, and time are *mks*. All other quantities, we will see, are combinations or derivations from these fundamental quantities. You must **ALWAYS** use units in your answers.

Powers of Ten in the SI Units

A decided advantage of the SI or mks system, compared to the British system (inches, slugs, etc.), is the use of powers of ten. In addition to the fundamental units (meter, kilogram, second) one can use prefixes to these units when that is more convenient. Some of these prefixes are given on pages 5–6, and you should memorize these. A more extended set of prefixes are is shown in the table below, taken from page A8 in Appendix F which as the complete set from 10^{-24} to 10^{+24}.

Power of 10	Prefix	Abbreviation
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E

Note the capitalization of the mega–, tera–, peta–, and exa– prefixes, while all the other prefixes, including all those with negative powers of ten, have lower case abbreviations. Typically, for derived units coming from a person's names such as volt (V) from Volta, or newton (N) from Issac Newton, these too will have capital letters in their abbreviations.

You should be familiar with *GBytes*, meaning 1 billion[1] bytes, as a unit of memory or disk space on a personal computer. It should not be too long before we see these quantities quoted in units of *TBytes*. In the high energy nuclear experiments where I work, we quote our data outputs in units of *PBytes*, which is pronounced as peta-Bytes.

[1] In British English a billion is what we would call a trillion in America.

Derived Quantity: Density

Besides the fundamental quantities of length, mass, and time, there are also many (many) so–called *derived quantities* which can be always be expressed in terms of the fundamental quantities. One will also be seeing derived quantities defined in terms of other derived quantities, but ultimately everything can be expressed as combinations of length mass and time. For now we look at examples of such quantities.

Density

Density is the mass of an object divided by its volume. If the object is composed entirely of one substance, such as iron or gold or water or nitrogen, then the density will be the same throughout the object. Density is usually given the Greek symbol ρ ("rho")

$$\rho = \frac{mass}{volume} = \frac{m}{V} \tag{1a}$$

A short table of densities of various substances is given on page 457. By knowing the density of a substance and the volume of the substance one can find the mass of the substance according to:

$$m = \rho V \tag{1b}$$

For example what is the mass of a solid cube of aluminum with a volume of 0.2cm^3 ? First realize that aluminum has a density of 2.7 gm/cm^3, and then use the formula (1b) above

$$m = \rho V = 2.7 \frac{\text{g}}{(\text{cm})^3} \cdot 0.2(\text{cm})^3 = 0.54 \text{ gm}$$

Finally, one can compute the number atoms N in the above cube by knowing that in *one mole* of a substance there are *Avogadro's* number of atoms:

1 Mole $\equiv$ Molecular Weight in Grams

Avogadro's Number (N_A) $\equiv 6.02 \text{ x } 10^{23}$ atoms

For aluminum 1 Mole = 27 grams, so:

$$\frac{N_A}{27 \text{ gm}} = \frac{N}{0.54 \text{ gm}} \Longrightarrow N = \frac{N_A \cdot 0.54 \text{ gm}}{27 \text{ gm}} = 1.2 \text{ x } 10^{22} \text{ atoms}$$

You should look carefully in the above equations to see how the units in the denominator and the numerator tend to cancel out such that you get the correct units in the final answer. We will explore this topic more in the following page.

Dimensional Analysis

It is important that you realize that all formulas and equations must be *dimensionally correct.* That is the left side must contain the same dimensions as the right side. Also, if you add two quantities in a formula, they must have the same basic dimensions.

For example we will look at the equation for distance traveled given an initial speed and a constant acceleration. First we have to be told that **speed** is defined as distance traveled divided by the time it took to travel that distance:

$$v \equiv \frac{distance}{time} \Longrightarrow meters/second \Longrightarrow LT^{-1}$$

Here L and T are the dimensions length and time which are treated as algebraic quantities.

Next, we have to be told that **acceleration** is defined as the change in speed divided by the time it took for that change to occur:

$$a \equiv \frac{\Delta v}{\Delta t} \Longrightarrow meters/(second)^2 \Longrightarrow LT^{-2}$$

Now the formula for the distance traveled $x(t)$ with an initial speed v_0 and a constant acceleration a:

$$x(t) = x_0 + v_0 \cdot t + \frac{1}{2} a \cdot t^2 \tag{2}$$

Dimensional Analysis of this equation

$$L = L + (LT^{-1}) \cdot T + \frac{1}{2}(LT^{-2}) \cdot T^2$$

$$L = L$$

So both sides of the equation are in terms of Length.

Dimensional Analysis Counter-Example

Contrast the correct Equation 2 with the following **incorrect** expression

$$x(t) = at$$

Dimensional Analysis of this equation

$$L = \ (?)\ (LT^{-2}) \cdot T$$
$$L \neq LT^{-1}$$

Units Conversion

Often you will be given a problem in one set of units, but in order to find the answer you must change to another set of units. For example, change the density of water from grams/cubic centimeter into kilograms/cubic meter

$$\rho_{water} = 1\frac{g}{(cm)^3}$$
$$\rho_{water} = \frac{10^{-3}kg}{(10^{-2}m)^3}$$
$$\rho_{water} = \frac{10^{-3}kg}{(10^{-6}m^3)}$$
$$\rho_{water} = 10^{+3}\frac{kg}{m^3}$$

One cubic centimeter of water contains one gram,

but

one cubic meter of water contains 1,000 kilograms!

CHAPTER 1: VECTORS

Most physical quantities are either **Scalars** or **Vectors**

A **scalar** is a physical quantity which can be specified by just giving the magnitude only, in appropriate units.

Examples of scalars are *mass, time, length, speed*.

Scalar quantities may be added by the normal rules of mathematics

---

A very important class of physical quantity is **Vectors**.

A vector is characterized by specifying both a magnitude (in the proper units) **AND** a direction.

Examples of vector quantities are *force, velocity, momentum.*

Vector quantities are added together by a special rule of *vector addition.*

There are two methods of doing *vector addition:*

1) Graphical addition (triangle, parallelogram, or polygon methods)
2) Analytic method — addition of the **vector components**
    a) first: **resolve** the vectors into their X and Y components
    b) second: add the vector X and Y components separately
    c) third: use the Pythagorean theorem to form the **resultant** vector

---

The **displacement vector** is the vector which characterizes the change in position of a particle.

There are two ways of **multiplying** two vectors:

1) The **scalar** or **dot** product generates a scalar $s \equiv \vec{A} \cdot \vec{B}$ of magnitude ABcos θ_{AB}
2) The **vector** or **cross** product generates a vector $\vec{C} \equiv \vec{A} \times \vec{B}$ whose magnitude C = ABsin θ_{AB} and whose direction is by the *right-hand rule.*

Vectors and Scalars

Some quantities in physics such as mass, length, or time are called *scalars*. A quantity is a scalar if it obeys the ordinary mathematical rules of addition and subtraction. All that is required to specify these quantities is a magnitude expressed in an appropriate units.

A very important class of physical quantities are specified not only by their magnitudes, but also by their *directions*. Perhaps the most important of these quantities is **FORCE**. Consider a heavy trunk on a smooth (almost slippery) floor, weighing say 100 pounds. You want to move the trunk but you are only able to lift 50 pounds.

What do you do?

A **vector** must always be specified by giving its magnitude and direction. In turn the vector's direction must be given with respect to some known direction such as the horizontal or the vertical direction, or perhaps with respect to some pre–defined "X" axis.

The specification of the magnitude and direction does not have to be direct or explicit. The specification can be indirect or implicit by giving the "X" and "Y" **components** of the vector, and it is up to you to use the Pythagorean theorem to calculate the actual magnitude and direction.
(Do you remember your trigonometry?)

1) What is a right triangle ? How many *degrees* are there in a triangle ?

2) What are the definitions of **sine**, **cosine**, and **tangent** ?

3) What is the Pythagorean theorem ?

4) What is the *law of sines* ?

5) What is the *law of cosines* ?

6) What is a *radian* ?

Vector Addition by Graphical Means

Depiction of Vectors

A vector is represented by an arrow (a line with an arrowhead).
The *length* of the line is in some proportion to the *magnitude* of the vector.
The *orientation* of the line reflects the *direction* of the vector

How do I know that this is a vector and not just another arrow?

Answer: If it's a vector, it must add like a vector

In order to add, I must have another vector. With two vectors, I can add them together to form a **RESULTANT**.

Two vectors, $\vec{A}$ and $\vec{B}$, can be added *graphically* by the simple triangle rule: Place the tail of the second vector at the head of the first vector, and then draw a line from the tail of the first vector to the head of the second vector. That line, both in magnitude and direction is the sum (Resultant) of the two original vectors.

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$

If there are more than two vectors to be added, say $\vec{A} + \vec{B} + \vec{C} + \vec{D}$, then the triangle rule is simply extended to the polygon rule. Just keep placing the tail of the next vector at the head of the preceding vector. The resultant is represented by a line from the tail of the first vector to the head of the last vector.

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{D}}$$

Properties of Vector Addition

1) Vector addition is commutative: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
2) Vector addition is associative: $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$

A *scalar* may multiply a vector *e.g.* $2\vec{\mathbf{A}}$. This produces a vector twice as large as the original vector, and in the same direction as the original vector. On the other hand $-0.5\vec{\mathbf{A}}$ produces a vector half the size of the original vector, and in the *opposite* direction to the original vector's direction.

Analytic Addition of Vectors using Vector Components

The graphical addition of vectors is not terribly convenient, especially if a numerical solution is required. Much more often you will have to add vectors *analytically*. By that is meant that you first *resolve* the vectors into their perpendicular components, then add the components by ordinary mathematics, and finally reconstitute the resultant with trigonometry and the Pythagorean theorem.

Resolving a vector into its perpendicular components.

Say that you are given a vector $\vec{\mathbf{A}}$ oriented at an angle θ with respect to the x (horizontal) axis. This original vector may be **resolved** into two perpendicular components, $\vec{\mathbf{A}}_{\mathbf{x}}$ and $\vec{\mathbf{A}}_{\mathbf{y}}$ which **replace** $\vec{\mathbf{A}}$.
In other words, the original vector no longer exists, and one has two mutually perpendicular vectors in its place.

The *magnitudes* of the two component vectors are given by:

$$A_x = A\cos\theta$$
$$A_y = A\sin\theta$$

The *directions* of the two component vectors are given by two **unit** vectors, $\vec{\mathbf{i}}$ and $\vec{\mathbf{j}}$ along the x and y directions respectively:

$$\vec{\mathbf{A}}_{\mathbf{x}} = A_x\vec{\mathbf{i}}$$
$$\vec{\mathbf{A}}_{\mathbf{y}} = A_x\vec{\mathbf{j}}$$

Clearly the above process can be run backwards. One can obtain back the original vector $\vec{\mathbf{A}}$ by using trigonometry:

For the *magnitude* use the Pythagorean theorem: $A = \sqrt{A_x^2 + A_y^2}$. For the *direction* use the right triangle trigonometry definitions: $\tan\theta = A_y/A_x \Longrightarrow \theta = \tan^{-1} A_y/A_x$

Analytic Addition of Vectors using Vector Components

The graphical addition of vectors is not terribly convenient, especially if a numerical solution is required. Much more often you will have to add vectors *analytically*. By that is meant that you first *resolve* the vectors into their perpendicular components, then add the components by ordinary mathematics, and finally reconstitute the resultant with trigonometry and the Pythagorean theorem.

Instead of one vector $\vec{\mathbf{A}}$, now take another vector $\vec{\mathbf{B}}$
Let's say $\vec{\mathbf{A}}$ is at an angle α, and $\vec{\mathbf{B}}$ is at an angle β
The vector sum of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is denoted by $\vec{\mathbf{R}}$

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$

This can be solved component–by–component

$$R_x = A_x + B_x$$
$$R_y = A_y + B_y$$

Now solve for $\vec{\mathbf{R}}$. First the *magnitude*

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

And now the *direction* of $\vec{\mathbf{R}}$ which we symbolize as γ

$$\gamma = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{(A_y + B_y)}{(A_x + B_x)}$$

Analytic Addition of Vectors using Vector Components

The graphical addition of vectors is not terribly convenient, especially if a numerical solution is required. Much more often you will have to add vectors *analytically*. By that is meant that you first *resolve* the vectors into their perpendicular components, then add the components by ordinary mathematics, and finally reconstitute the resultant with trigonometry and the Pythagorean theorem.

TABLE FORM OF ANALYTIC ADDITION

Vector	Angle	Magn. of x component	Magn. of y component
$\vec{\mathbf{A}}$	α	$A\cos\alpha$	$A\sin\alpha$
$\vec{\mathbf{B}}$	β	$B\cos\beta$	$B\sin\beta$
$\vec{\mathbf{R}}$	$\gamma = \tan^{-1}\frac{R_y}{R_x}$	$R_x = A\cos\alpha + B\cos\beta$	$R_y = A\sin\alpha + B\sin\beta$

Worked Example

A hiker walks 25 km due southeast ($= -45^{\mathrm{O}}$) the first day, and 40 km at 60^{O} north of east. What is her total displacement for the two days?

Arrange the problem in the table above with $\vec{\mathbf{A}}$ being the first day's displacement and $\vec{\mathbf{B}}$ being the second day's displacement:

Vector (km)	Angle ($^{\mathrm{O}}$)	Magn. of x component (km)	Magn. of y component (km)
A = 25	-45	$A\cos(-45) =$	$A\sin(-45) =$
B = 40	$+60$	$B\cos(+60) =$	$B\sin(+60) =$
R =	$\gamma = \tan^{-1}\frac{R_y}{R_x} =$	$R_x =$	$R_y =$

Analytic Addition of Vector Components

Worked Example

You are given a displacement of 20 km to the West, and a second displacement at 10 km to the North. What is the sum of the two displacements?

Vector (km)	Angle ($^{\text{O}}$)	Magn. of x component (km)	Magn. of y component (km)
Vector	Angle	Magn. of x component	Magn. of y component
(km)	($^{\text{O}}$)	(km)	(km)
A = 20	+180	$A\cos(+180) =$	$A\sin(+180) =$
B = 10	+90	$B\cos(+90) =$	$B\sin(+90) =$
R =	$\gamma = \tan^{-1}\frac{R_y}{R_x} =$	$R_x =$	$R_y =$

WARNING Know your quadrants !!
