Chapter 16: Quantitative Measures for Sound Speed of Sound in Gases, Liquids, and Solids

The speed of sound in air, at standard temperature and pressure, is 344 m/s (see speeds of sound in Table 16.1, page 534). In general for an ideal gas the formula for the speed of sound is

Gas:
$$v = \sqrt{\frac{\gamma RT}{M}}$$

where R is the ideal gas constant which we will see shortly, γ is a ratio of heat capacities (=1.40 for diatomic gases, =1.67 for the monotonic noble gases), T is the temperature in Kelvin, and M is the molecular weight of the gas in moles. For example, air has an average $M = 28.8 \times 10^{-3}$ kg/mol, room temperature T = 293 K, and R = 8.314 in these MKS units. So the speed of sound in air is

$$v_{\text{air}} = \sqrt{\frac{(1.40)(8.314)(203)}{28.8 \times 10^{-3}}} = 344 \text{ m/s}$$

For liquids, the speed of sound depends on the bulk modulus B of the liquid and the density ρ of the liquid

Liquid:
$$v = \sqrt{\frac{B}{\rho}}$$
 For water the speed of sound is 1543 m/s

For solids, the speed of sound depends on the Young's modulus Y, and the density of the solid

Solid:
$$v = \sqrt{\frac{Y}{\rho}}$$
 For steel, the speed of sound is 5941 m/s

Intensity of Sound

As with energy in any wave, the intensity of sound various with the square of the amplitude of the wave. In the case of sound, there is a *dimensionless* measure of sound intensity, which essentially is the ratio of a given intensity to the minimum intensity detectable by the human ear. This minimum intensity number is taken as $I_0=10^{-12}$ W/m². The sound intensity (ratio) is called the *decibel* and is symbolized as β . The decibel definition for a sound intensity I is

$$\beta \equiv 10 \log_{10} \frac{I}{I_0}$$
 See Table 16.2, page 540, for typical sound intensities

Chapter 17: Temperature and Heat

Temperature

Temperature is a **new physical quantity**, for which there are three scales in common use: Fahrenheit (F), Celsius (C), and Kelvin (K, or absolute temperature). These three scales are all related to each other:

$$T_F = \frac{9}{5}T_C + 32$$
 and $T_K = T_C + 273.15$ Note: $-40^{\circ} \text{ C} = -40^{\circ} \text{ F}$

At the atomic or molecular level, we can discover that temperature is a measure of the *average kinetic energy* in the atoms or molecules.

Thermal expansion (see Tables 17.2 and 17.3 on page 578)

Solids and liquids will generally expand when the temperature rises. The expansion is proportional to the amount of temperature change according to

 $\Delta L = \alpha L_0 \Delta T \quad \text{and} \quad \Delta V = \beta L_0 \Delta T$

For solids $\beta = 3\alpha$. If a solid has linear dimension L_0 , and it is heated but constrained to remain at the same length, then the solid will be under a stress

$$\frac{F}{A} = -Y\alpha\Delta T$$

Liquid volume expansion coefficients are ≈ 10 times larger than those of a solid.

Heat (See Table 17.4 on page 587)

Heat is a unit of energy, and can be quoted in *calories* where 1 calorie=4.186 Joules. A solid or liquid of mass m will increase its temperature by an amount ΔT if an amount heat Q is added according to the formula

$$Q = mc\Delta T$$

The quantity c is called the *heat capacity* of the material, and is typically quoted in calories/gram-K, or Joules/gram-K. Values of c for different materials are given in Table 17.3, on page 585. A solid of mass m can change to a liquid (melting), at a fixed temperature, with an amount of heat

 $Q = mL_f$ $L_f \equiv$ Latent heat of fusion

A liquid of mass m can change to a gas (evaporating), at a fixed temperature, with an amount of heat

$$Q = mL_v$$
 $L_v \equiv$ Latent heat of vaporization

Chapter 17: Heat Transfer

Origin of the calorie unit

A few hundred years ago when people were investigating heat and temperature phenomena, the idea was that heat was some kind of invisible *caloric* fluid which could flow from one material to another. This is the origin of the word calorie, which we now know as a form of energy. A calorie can be defined as the amount of heat energy needed to raise the temperature of water by one Celsius degree, or one Kelvin.¹ Since the heat capacity formula for raising the temperature of a mass is given by

 $Q = mc\Delta T$ (Heat capacity "c" is also called specific heat)

this means that the heat capacity number c for water is 1 calorie/gram-K.

Methods of heat transfer

Heat can be transferred by three methods

1) Conduction. If we have a material of cross-sectional area A and length L, with one end of the length at a high temperature T_H and the other length at a low temperature T_L , then heat will flow from the high temperature side to the lower temperature side according to:

$$H = \frac{dQ}{dt} = kA\frac{T_H - T_L}{L}$$

Here k is the thermal conductivity of the material in units of W/m-K.

- 2) Convection. Convection involves masses with high temperature moving into regions of low temperature. Rising currents of air from the Earth's surface are examples of heat transfer by convection. There is no simple equation for convection as there is for conduction.
- 3) Radiation. Radiation is the emission of heat energy in the form of electromagnetic waves from their surface. All objects above absolute zero temperature will radiate according to the Stefan-Boltzmann equation

$$H = Ae\sigma T^4$$

where A is the area of the body's surface, e is the emissivity parameter which is dimensionless and is $0 < e \leq 1$, σ is the Stefan-Boltzmann constant with a value $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{-K}^4$, and T is temperature in the Kelvin scale.

¹The modern language convention is to simply say Kelvin, and not Kelvin degree or degrees Kelvin.

Calorimetry Fundamentals

Basic Equation

Calorimetry problems deal with the transfer of heat from a hot system at temperature T_H to a one or more colder systems at temperature T_C . Because of energy conservation there is an exact equality

Heat Lost by Hot System = Heat Gained by Colder System(s)

Typically in such problems a final equilibrium temperature T_E is reached where the final temperature is between the two original temperatures: $T_C < T_E < T_H$.

Heat Loss and Heat Gain Mechanisms

Heat is lost by a hot system of mass m_a in the following two basic ways

1) Lowering of the temperature of the hot system by an amount ΔT_a , for which the heat loss ΔQ_a is proportional to the specific heat

 $\Delta Q_a = m_a c_a \Delta T_a \qquad (\Delta T_a \text{ is negative for heat loss})$

2) Condensing (reverse of evaporation) or freezing (reverse of melting) for which the heat loss is

$$\Delta Q_{av} = -m_a L_v \quad \text{or} \quad \Delta Q_{af} = -m_a L_f$$

Notice that the condensing or freezing occurs at a specific temperature characteristic of the material m_a , such as 100^o C for condensing stream or 0^o C for freezing water.

Heat is gained by a colder system of mass m_b in two comparable ways

1) Raising of the temperature of the colder system by an amount ΔT_b , for which the heat gain ΔQ_b is proportional to the specific heat

 $\Delta Q_b = m_a c_b \Delta T_b \qquad (\Delta T_b \text{ is positive for heat gain})$

2) Melting or evaporation for which the heat gain is

$$\Delta Q_{bf} = +m_b L_f \quad \text{or} \quad \Delta Q_{av} = +m_a L_v$$

Again the melting or evaporation occurs at a constant temperature characteristic of the material m_b .

Calorimetry Example

Example 17.9

A student wants to cool 0.25 kg (m_a) of diet soda (mostly water) initially at 25° C. This will be done by adding ice cubes which are at -20° C coming out of the freezer. What mass m_b of ice must be added in order that the diet soda plus melted ice all be at a final temperature of 0° C?

In this problem the mostly water diet soda will cool down from the initial 25° C to the final temperature $T_E = 0^{\circ}$ C. This will be a heat loss Q_a

While this is happening the ice will first go from -20° C to 0° C, gaining an amount of heat Q_b . Then all the ice will melt, which will mean a further gain of heat Q_{bf} by the ice.

We have the heat energy conservation equation

$$Q_a + Q_b + Q_{bf} = 0$$

where the heat lost by the soda is

$$Q_a = m_a c_a \Delta T_a = (0.25 \text{ kg})(4190 \text{ J/kg-K})(0^{\text{O}} - 25^{\text{O}}) = -26,000 \text{ J}$$

while the heat gained in raising the temperature of the ice is

$$Q_b = m_b c_b \Delta T_b = (m_b \text{ kg})(2100 \text{ J/kg-K})(0^{\text{O}} - (-20)^{\text{O}}) = m_b(4.2 \times 10^4 \text{ J/kg})$$

and lastly the heat gained by melting the ice is

$$Q_{bf} = m_b L_f = m_b (3.34 \times 10^5 \text{ J/kg})$$

So we get the equation

$$-26,000 \text{ J} + m_b \left[(4.2 \times 10^4 \text{ J/kg}) + (3.34 \times 10^5 \text{ J/kg}) \right]$$
$$\implies m_b = 0.069 \text{ kg}$$

You should note two features of this answer. The mass of ice needed is about 28% of the original mass of the soda, which sounds reasonable. Second, the major fraction of the heat gained by the ice is in the melting phase. You can see that this part of the heat gain, which is the second term in the brackets above, is almost 8 times that of the first term which represents raising the temperature of the ice itself.

Further thought question: How would you modify this solution if you wanted to have the final temperature of the soda and the melted ice to be at 1° C instead of 0° C?

Thermal Conduction

Heat Flow Model

Thermal conduction through a material is the simplest type of heat transfer to model, and it follows closely the old "caloric fluid" idea. You can think of the heat as something in a hot material which is flowing downhill into a colder material. In reality, since temperature is a measure of the average kinetic energy of a material, what is happening is that kinetic energy is being transferred from a hot material to a colder material.

Heat Conductivity Equation

The conduction of heat, H in Joules/second, through a cross-sectional area A over a length L between a hot region T_H and a colder region T_C can be calculated with the heat conductivity equation

$$H = \frac{dQ}{dt} = kA\frac{T_H - T_C}{L}$$

where k is the thermal conductivity constant of the material. Values of k for different materials are given in Table 17.5 on page 592. Materials with very low heat conductivities, such as styrofoam or fiberglass, are actually good heat insulators. On the other hand, metals are good heat conductors especially copper and silver.

Heat Conductivity Example 17.13

Heat conductivity problems typically have a region of high temperature "reservoir" which stays at a constant T_H . Similarly there is a region of colder temperature which is at a constant T_C . In between these two regions there is a heat conducting material of end cross sectional area A and length L through which heat is transferred from the hot reservoir to the colder reservoir. No heat is lost the sides of the heat conducting material, but heat enters at one end and exits the other end in a steady-state manner.

In this example a steel bar 10.0 cm long with cross-sectional area $A = 4 \text{ cm}^2$ has one end in a steam bath at $T_H = 100^{\circ}$ C. The other end of the steel bar is welded to a copper bar of the same cross sectional area but 20.0 cm long. The free end of the copper bar is in an ice-water mixture kept at $T_C = 0^{\circ}$ C. What is the heat flow H through these bars, and what is the temperature T_M where the two bars are welded together?

Chapter 18: Thermal Properties of Matter Ideal Gas Law

For an ideal gas the pressure P, the volume V, and the absolute temperature T are related by the ideal gas law

$$PV = nRT$$

where n is the number of moles of the gas, and R is the ideal gas constant. In the MKS system, R = 8.314 J/mole-K. If pressure is given in atmospheres and volume in liters, then R = 0.08206 L-atm/mol-K. From these two expressions for R, it should be obvious that a "liter-atmosphere" has dimensions of energy (or work).

Kinetic Theory of Ideal Gases

The microscopic physics explanation for the ideal gas law is the Kinetic Theory. This theory has four assumptions: 1) a volume V of a gas contains a very large (Avogadro scale) number of identical molecules each of mass m; 2) the molecules behave as point particles and don't interact with each other; 3) the molecules collide elastically with the walls of the container according to Newton's Laws; and 4) the pressure developed on the walls comes from these collisions since the walls are consider to be perfectly rigid and immobile.

Based on these four assumptions, it is possible to derive the ideal gas law and to get a microscopic interpretation of temperature. Specifically we can derive that the average kinetic energy of a molecule with mass m in an ideal gas is proportional to the absolute temperature of the gas

$$\frac{1}{2}mv_{\text{average}}^2 = \frac{3}{2}kT$$

The constant k is called the Boltzmann constant and has a value $k = 1.381 \times 10^{-23}$ J/molecule-K. The Boltzmann constant is defined as the ideal gas constant R divided by Avogadro's number N_A

$$k \equiv \frac{R}{N_A} = \frac{8.314 \text{J/mol-K}}{6.022 \times 10^{23} \text{molecules/mol}} = 1.381 \times 10^{-23} \text{ J/molecule-K}$$

Consequences of the Kinetic Theory of Gases Root-Mean-Square Speed of Molecules in an Ideal Gas

The root-mean-square speed v_{rms} for molecules in an ideal gas turns out to be:

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$$

where $M = N_A m$ which is the molecular weight of one mole of a gas since there are Avogadro's number of molecules in one mole. You can now see why the Earth's atmosphere is missing a light gas such as helium. The average speed of a helium molecule in the Earth's atmosphere, given the temperature T in the Earth's atmosphere, would be bigger than the escape velocity of the Earth's gravity. Hence any helium in the Earth's atmosphere is bound to disappear into outer space. Fortunately for us humans, the average speed of an oxygen molecule, at the same temperature T, is much slower (by what factor?) than that of helium, and slower than the escape velocity of the Earth's gravity. So the Earth has retained these heavier gas molecules over billions of years.

Mean Free Path

The kinetic theory of molecules in a gas assumes that the molecules do not have inelastic interactions with each other. The molecules can collide with each other elastically, and that is how two gases reach an equilibrium temperature.

On pages 624–625 in the text book, there is a short derivation of the *mean-free*path formula. The mean-free-path, with the symbol λ is the average distance that a molecule can travel in a gas before it collides elastically with a second molecule. This λ distance depends on the total number of molecules in the gas volume, and the radius r of the molecules which are taken as spheres:

$$\lambda = \frac{kT}{4\pi\sqrt{2}r^2p}$$

where T is the temperature of the gas and p is the pressure of the gas. From the ideal gas equation you know that T/p is directly proportional to the number of moles in the gas. For air at standard temperature and pressure, you can work out (example 18.8 on page 625) that the mean free path is 5.8×10^{-8} m, as compared with the size of the oxygen molecule which has a radius 2.0×10^{-10} m. The mean free path is almost 300 times larger than the radius of the molecule. You can also work out that the $v_{rms} = 484$ m/s, meaning that in one second there will be 10^{10} collisions. That is 10^{12} times smaller than Avogadro's number.

Kinetic Theory and the Heat Capacities of Gases Monotomic Gases

Since we know that the average kinetic energy of a gas molecule is directly proportional to the temperature, it is easy to predict the heat capacities of ideal gases (see page 626). For monotomic gases, which are essentially point particles, the heat capacity is given simply by

Monotomic gas molecule:
$$C_V = \frac{3}{2}R$$

This (upper case notation) means the number of Joules required to increase the temperature of one mole of the gas by one K (or Celsius degree), as long as the gas is kept at constant volume. If the gas is allowed to expand at constant pressure, there is another heat capacity C_P which we will study in Chapter 19.

Diatomic Gases

Diatomic gases, consisting of two molecules such as O_2 , can also rotate about their center-of-mass. So adding heat to a diatomic molecule can also produce rotational energy as well as increase the translational energy. Since only the translational energy is proportional to the temperature, then there is a different value for the heat capacity of a diatomic molecule. Nonetheless, there is still a simple prediction from the kinetic theory

Diatomic gas molecule:
$$C_V = \frac{5}{2}R$$

We will be using the molar heat capacities of gases in our studies of the first and second law of thermodynamics.