(REVIEW) Chapter 17: Heat Transfer Origin of the calorie unit

A few hundred years ago when people were investigating heat and temperature phenomena, the idea was that heat was some kind of invisible *caloric* fluid which could flow from one material to another. This is the origin of the word calorie, which we now know as a form of energy. A calorie can be defined as the amount of heat energy needed to raise the temperature of water by one Celsius degree, or one Kelvin.¹ Since the heat capacity formula for raising the temperature of a mass is given by

 $Q = mc\Delta T$ (Heat capacity "c" is also called specific heat)

this means that the heat capacity number c for water is 1 calorie/gram-K.

Methods of heat transfer

Heat can be transferred by three methods

1) Conduction. If we have a material of cross-sectional area A and length L, with one end of the length at a high temperature T_H and the other length at a low temperature T_L , then heat will flow from the high temperature side to the lower temperature side according to:

$$H = \frac{dQ}{dt} = kA\frac{T_H - T_L}{L}$$

Here k is the thermal conductivity of the material in units of W/m-K.

- 2) Convection. Convection involves masses with high temperature moving into regions of low temperature. Rising currents of air from the Earth's surface are examples of heat transfer by convection. There is no simple equation for convection as there is for conduction.
- 3) Radiation. Radiation is the emission of heat energy in the form of electromagnetic waves from their surface. All objects above absolute zero temperature will radiate according to the Stefan-Boltzmann equation

$$H = Ae\sigma T^4$$

where A is the area of the body's surface, e is the emissivity parameter which is dimensionless and is $0 < e \leq 1$, σ is the Stefan-Boltzmann constant with a value $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{-K}^4$, and T is temperature in the Kelvin scale.

¹The modern language convention is to simply say Kelvin, and not Kelvin degree or degrees Kelvin.

The Mechanical Equivalent of Heat

Heat energy stored in food is counted in kilocalories or Calories (upper case !) where 1 Calorie = 4,186 Joules.

Example

A student eats a dinner rated a 2,000 Calories, and the student decides to "burn that off" by constantly lifting a 50 kg mass (110 lbs) 2 meters high. How many times would the student have to lift such a mass in order to burn off all that dinner? Solution

The amount of energy contained in the dinner is given by

 $2000 \cdot 4,186 = 8.37 \ge 10^7$ Joules

Lifting a 50 kg mass by 2 meters requires an energy

$$E = mgh = 50 \cdot 9.8 \cdot 2 = 980$$
 Joules

So the total number of times to burn off the dinner is

$$n = \frac{8.37 \ge 10^7}{980} = 8,840$$

Assuming it takes 5 seconds to lift weight each time, then the student would need 12 *hours* total to successfully burn off the dinner.

Latent Heats

A more complicated type of problem involves a phase change. Suppose steam at 130° C is chosen to warm 200 g of water in a 100 g glass container from 20 to 50° C. How much mass of steam would be needed. To solve this problem you need to realize that the steam first cools down to 100° C, and then at 100° it loses its heat of fusion (at 3.33 J/kg with no temperature change), and finally it cools down as water until an equilibrium temperature is reached. Now while the steam is losing heat, the glass and the water inside are gaining heat.

You should realize that when a substance melts, it absorbs a certain number of calories per gram called the Latent Heat of Fusion. When that substance freezes, it gives up that same number of calories per gram. Similarly, when a substance boils or evaporates, it absorbs a certain number of calories per gram called the Latent Heat of Vaporization. Conversely when the substance condenses out of the gas phase back into the liquid phase, that substance gives up this Latent Heat of Vaporization. Both melting (freezing) and boiling (condensing) occur without any temperature change.

(REVIEW Chapter 17) Thermal Conduction Heat Flow Model

Thermal conduction through a material is the simplest type of heat transfer to model, and it follows closely the old "caloric fluid" idea. You can think of the heat as something in a hot material which is flowing downhill into a colder material. In reality, since temperature is a measure of the average kinetic energy of a material, what is happening is that kinetic energy is being transferred from a hot material to a colder material.

Heat Conductivity Equation

The conduction of heat, H in Joules/second, through a cross-sectional area A over a length L between a hot region T_H and a colder region T_C can be calculated with the heat conductivity equation

$$H = \frac{dQ}{dt} = kA\frac{T_H - T_C}{L}$$

where k is the thermal conductivity constant of the material. Values of k for different materials are given in Table 17.5 on page 592. Materials with very low heat conductivities, such as styrofoam or fiberglass, are actually good heat insulators. On the other hand, metals are good heat conductors especially copper and silver.

Heat Conductivity Example 17.13

Heat conductivity problems typically have a region of high temperature "reservoir" which stays at a constant T_H . Similarly there is a region of colder temperature which is at a constant T_C . In between these two regions there is a heat conducting material of end cross sectional area A and length L through which heat is transferred from the hot reservoir to the colder reservoir. No heat is lost the sides of the heat conducting material, but heat enters at one end and exits the other end in a steady-state manner.

In this example a steel bar 10.0 cm long with cross-sectional area $A = 4 \text{ cm}^2$ has one end in a steam bath at $T_H = 100^{\circ}$ C. The other end of the steel bar is welded to a copper bar of the same cross sectional area but 20.0 cm long. The free end of the copper bar is in an ice-water mixture kept at $T_C = 0^{\circ}$ C. What is the heat flow H through these bars, and what is the temperature T_M where the two bars are welded together?

Chapter 18: Thermal Properties of Matter Ideal Gas Law

For an ideal gas the pressure P, the volume V, and the absolute temperature T are related by the ideal gas law

$$PV = nRT$$

where n is the number of moles of the gas, and R is the ideal gas constant. In the MKS system, R = 8.314 J/mole-K. If pressure is given in atmospheres and volume in liters, then R = 0.08206 L-atm/mol-K. From these two expressions for R, it should be obvious that a "liter-atmosphere" has dimensions of energy (or work).

Kinetic Theory of Ideal Gases

The microscopic physics explanation for the ideal gas law is the Kinetic Theory. This theory has four assumptions: 1) a volume V of a gas contains a very large (Avogadro scale) number of identical molecules each of mass m; 2) the molecules behave as point particles and don't interact with each other; 3) the molecules collide elastically with the walls of the container according to Newton's Laws; and 4) the pressure developed on the walls comes from these collisions since the walls are consider to be perfectly rigid and immobile.

Based on these four assumptions, it is possible to derive the ideal gas law and to get a microscopic interpretation of temperature. Specifically we can derive that the average kinetic energy of a molecule with mass m in an ideal gas is proportional to the absolute temperature of the gas

$$\frac{1}{2}mv_{\text{average}}^2 = \frac{3}{2}kT \Longrightarrow v_{\text{average}} = \sqrt{\frac{3kT}{m}}$$

The constant k is called the Boltzmann constant and has a value $k = 1.381 \times 10^{-23}$ J/molecule-K. The Boltzmann constant is defined as the ideal gas constant R divided by Avogadro's number N_A

$$k \equiv \frac{R}{N_A} = \frac{8.314 \text{J/mol-K}}{6.022 \times 10^{23} \text{molecules/mol}} = 1.381 \times 10^{-23} \text{ J/molecule-K}$$

Consequences of the Kinetic Theory of Gases Root-Mean-Square Speed of Molecules in an Ideal Gas v_{rms}

The root-mean-square speed v_{rms} for N molecules in an ideal gas turns out to be:

$$v_{rms} \equiv \sqrt{\frac{\sum_{i=1}^{N} v_i^2}{N}} = \sqrt{v_{average}^2} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$$

where $M = N_A m$ which is the molecular weight of one mole of a gas since there are Avogadro's number of molecules in one mole. You can now see why the Earth's atmosphere is missing a light gas such as helium. The average speed of a helium molecule in the Earth's atmosphere, given the temperature T in the Earth's atmosphere, would be bigger than the escape velocity of the Earth's gravity. Hence any helium in the Earth's atmosphere is bound to disappear into outer space. Fortunately for us humans, the average speed of an oxygen molecule, at the same temperature T, is much slower (by what factor?) than that of helium, and slower than the escape velocity of the Earth's gravity. So the Earth has retained these heavier gas molecules over billions of years.

Mean Free Path

The kinetic theory of molecules in a gas assumes that the molecules do not have inelastic interactions with each other. The molecules can collide with each other elastically, and that is how two gases reach an equilibrium temperature.

On pages 624–625 in the text book, there is a short derivation of the *mean-free*path formula. The mean-free-path, with the symbol λ is the average distance that a molecule can travel in a gas before it collides elastically with a second molecule. This λ distance depends on the total number of molecules in the gas volume, and the radius r of the molecules which are taken as spheres:

Mean Free Path:
$$\lambda = \frac{kT}{4\pi\sqrt{2}r^2p}$$

where T is the temperature of the gas and p is the pressure of the gas. From the ideal gas equation you know that T/p is directly proportional to the number of moles in the gas. For air at standard temperature and pressure, you can work out (example 18.8 on page 625) that the mean free path is 5.8×10^{-8} m, as compared with the size of the oxygen molecule which has a radius 2.0×10^{-10} m. The mean free path is almost 300 times larger than the radius of the molecule. You can also work out that the $v_{rms} = 484$ m/s, meaning that in one second there will be 10^{10} collisions. That is 10^{12} times smaller than Avogadro's number.

Kinetic Theory and the Heat Capacities of Gases Monatomic Gases

Since we know that the average kinetic energy of a gas molecule is directly proportional to the temperature, it is easy to predict the heat capacities of ideal gases (see page 626). For monatomic gases, which are essentially point particles, the heat capacity is given simply by

Monatomic gas molecule:
$$C_V = \frac{3}{2}R$$

This (upper case notation) means the number of Joules required to increase the temperature of one mole of the gas by one K (or Celsius degree), as long as the gas is kept at constant volume. If the gas is allowed to expand at constant pressure, there is another heat capacity C_P which we will study in Chapter 19.

Diatomic Gases

Diatomic gases, consisting of two molecules such as O_2 , can also rotate about their center-of-mass. So adding heat to a diatomic molecule can also produce rotational energy as well as increase the translational energy. Since only the translational energy is proportional to the temperature, then there is a different value for the heat capacity of a diatomic molecule. Nonetheless, there is still a simple prediction from the kinetic theory

Diatomic gas molecule:
$$C_V = \frac{5}{2}R$$

We will be using the molar heat capacities of gases in our studies of the first and second law of thermodynamics.

Chapter 19: Heat and the First Law of Thermodynamics

We have seen how heat is a form of energy which can be transferred from one substance to another when there is a temperature difference. We have also seen that there is a conservation of total energy law in mechanics which includes the heating effects of friction. This mechanics law is part of a very general law of conservation of energy which goes under the name of **The First Law of Thermodynamics**. This law has many formulations, but in this chapter it concerns the total internal energy U_{int} of a system, and how that total energy changes when heat ΔQ flows into the system and the system does work W during the same time

$$\Delta U_{\rm int} = Q - W \tag{19.5}$$

This equation states that the change in internal (total) energy of a system is equal to the heat added less the work done by that system in the process. The symbol U was used in mechanics to mean Potential Energy. In thermodynamics U_{int} means the *internal energy* of a system, and only changes in internal energy are significant. The subscript "int" further emphasizes the thermodynamic meaning.

Thermodynamics is a very important subject because the civilized world essentially runs on converting heat (from burning fuels) into work (making electricity, running cars, etc.) The simplest system to consider is that of n moles of a gas for which we have a definite volume V, pressure P, and temperature T

Ideal Gas Law: PV = nRT

A gas can do work by expanding against a piston, for example in the cylinders of a car engine. In considering how a gas can be affected, we define several types of processes or operations:

An adiabatic process is one in which no heat is transferred (Q = 0).

An isothermal process is one in which the temperature remains constant.

An isobaric process is one in which the pressure remains constant.

A **cyclic** process is one in which the gas goes through a series of pressure, volume, and temperature changes but finally comes back to the same initial conditions when the **cycle** is over.

The Work done by an Expanding Volume of Gas: $V_1 \rightarrow V_2$

Important General Formula for Work: $W = \int_{V_1}^{V_2} P \, dV$

The Work Done by an Expanding Gas

An expanding gas does work. This is a fundamentally important concept and is easy to verify. We know already that work is force times distance. So consider a piston on a gas container which moves out a distance dx when the gas pressure is P. This represents a net work on the piston

$$dW = Fdx = (PA)dx = P(Adx) = PdV$$

where dV is the volume increase in the gas when it expands. Now if the gas goes from some initial volume V_i to some final volume V_f , the net work is the integral of the dW

$$W = \int dW = \int_{V_i}^{V_f} P \, dV$$

This is often represents on a P vs V diagram with P as the y axis and V along the x axis. The work is then the area on the P vs. V curve.

Result for Isothermal Work

As it stands, we cannot do anything with the work equation unless we are told something about the process of expansion. One possibility is an *Isothermal Expansion* for which the temperature is held constant. If that is the case we can use the Ideal Gas Law to substitute for P in terms of V and the constant T

$$W = \int_{V_i}^{V_f} P \, dV = \int_{V_i}^{V_f} \frac{nRT}{V} \, dV$$
$$W = nRT \int_{V_i}^{V_f} \frac{1}{V} \, dV = nRT \left[\ln V \right]_{V_i}^{V_f} = nRT \ln \left(\frac{V_f}{V_i} \right)$$

Again, the important point to realize about the above formula for work is that *it applies only to an isothermal expansion or contraction !*.

The Internal Energy of a Gas

In the example of Isothermal Expansion, the gas does work but stays at constant temperature. Normally when a gas expands, it will cool down, but in this case we have specified that the temperature remain constant. The only way that can happen is that heat is added to the gas. If you like, consider that the gas container in the figure is on some kind of stove top and heat is constantly being added as the volume expands.

We can quantify this process by introducing the internal energy U_{int} of a gas system. The thermodynamic variable U_{int} is like the potential energy U_{int} in mechanics. Its absolute value is not important, only changes in U_{int} are physically significant. The variable U_{int} can be changed by the gas doing work $(U_{\text{int}} \text{ decreases})$, or heat being added to the gas $(U_{\text{int}} \text{ increases})$. We write

$$\Delta U_{\text{int}} = \Delta Q - W$$

The variable U_{int} depends only on the temperature of the gas. As long as the temperature remains constant, then U_{int} does not change².

Adiabatic Process

In an isothermal expansion, heat ΔQ must be added to the gas to do work. At the other extreme is the *adiabatic* process where no heat is added to the expanding gas. In this case the internal energy U_{int} must decrease by an amount equal to the work done by the expanding gas

$$Q = 0$$
 adiabatic process $\implies \Delta U_{\text{int}} = -W$

In an adiabatic process, the temperature of the gas must be lowered during an expansion. Generally, processes which happen so fast that there is no time for heat to be transferred (such as the compression stroke of a diesel engine, or the expansion of hot gases in a gasoline engine) are examples of adiabatic processes. Another example would be a well–insulated system such as the gas liquefaction in a refrigerator.

²The textbook uses only the symbol U for internal energy. I have added a subscript "int" in order to emphasize that U_{int} represents the internal energy in a thermodynamic system.

Heat Capacities for an Ideal Gas Heat Capacity Definitions and Relations

The specific heat or heat capacity c of a solid or a liquid was defined according to how much heat Q was needed to raise a given mass m of the substance by one K (or one Celsius degree):

$$Q = mc\Delta T$$

For an ideal gas, it is more useful to use either the 1) the molar heat capacity at constant volume called C_V , or 2) the molar heat capacity at constant pressure called C_P . We shall discover universal values for C_V and a universal equation relating C_P to C_V .

The molar heat capacity at constant volume C_V for a gas is defined according to the differential amount of heat dQ needed increase n moles of a gas by an amount dT in Kelvin at constant volume of the gas:

$$dQ = nC_V dT 19.12$$

Since the volume of the gas does not change, then all of this heat increases the internal energy of the gas

$$dU_{\text{int}} = nC_V dT$$

The molar heat capacity at constant pressure C_P for a gas is defined according to the differential amount of heat dQ needed increase n moles of a gas by an amount dT in Kelvin while the gas expands by a volume dV while the pressure p of the gas remains constant:

$$dQ = nC_p dT 19.14$$

Now while the gas expands at constant pressure, there is a differential amount of work done dW = pdV = nRdT. Since by conservation of energy (first law of thermodynamics) we have

$$dQ = dU_{\text{int}} + dW = nC_V dT + nRdT = nC_p dT$$

$$19.16$$

then we can see directly that

Universal relation for ideal gas molar heat capacities: $C_p = C_V + R$ 19.17

Molar Heat Capacities for Ideal Gases

Monatomic Gas Result

For monatomic gases, the Kinetic Theory predicts that $C_V = 3R/2$. By the universal ideal gas result we have

$$C_p = C_V + R = \frac{3}{2}R + R = \frac{5}{2}R$$

An important parameter for an ideal gas is the ratio C_p/C_V which is symbolized as γ

$$\gamma \equiv \frac{C_p}{C_V} = \frac{5/2R}{3/2R} = \frac{5}{3}R \approx 1.67R$$

Diatomic Gas Result

For diatomic gases, the Kinetic Theory predicts that $C_V = 5R/2$. By the universal ideal gas result we have

$$C_p = C_V + R = \frac{5}{2}R + R = \frac{7}{2}R$$
$$\gamma \equiv \frac{C_p}{C_V} = \frac{7/2R}{5/2R} = \frac{7}{5}R = 1.40R$$

Adiabatic Ideal Gas Law

The parameter γ is an important one for the **adiabatic ideal gas law**. You already know the ideal gas law, relating three parameters: p, V, and T as:

pV = nRT

If a gas is undergoing an *adiabatic change*, we can prove (see pages 662-663) the following adiabatic ideal gas law

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$
 or $p V^{\gamma} = \text{Constant}$ 19.23

As you can see, the adiabatic gas law relates two of the three parameters. You can predict the third via the regular ideal gas law. You can re-write the adiabatic gas law in terms of T and V as

$$T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$$
 or $T V^{\gamma - 1} = \text{Constant}$

In the above formulas, the change in the state of the gas is adiabatic, meaning that no heat entered or left the gas.

Work Done During an Adiabatic Gas Expansion Using the adiabatic ideal gas law

We use the adiabatic ideal gas law to get an expression for the work done by a gas in going from a state (p_1, V_1) to a state (p_2, V_2) in an adiabatic change:

$$pV^{\gamma} = C \Longrightarrow p = \frac{C}{V^{\gamma}}$$

$$W_{1\to 2} = \int_1^2 p dV = C \int_{V_1}^{V_2} \frac{dV}{V^{\gamma}} = C \frac{1}{\gamma - 1} \left(\frac{1}{V_1^{\gamma - 1}} - \frac{1}{V_2^{\gamma - 1}} \right)$$

Since the constant $C = p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$ we will get a final simple result as

Work done in an adiabatic gas change:
$$W_{1\rightarrow 2} = \frac{1}{\gamma - 1} (p_1 V_1 - p_2 V_2)$$

Example of Adiabatic Compression and Work, page 697

The piston in a car's diesel engine compresses the volume a fuel-air mixture by a factor of 15. Suppose the initial pressure is one atmosphere $(1.01 \times 10^5 \text{ Pa})$, and the initial temperature is 27° C (300 K). Find the final pressure and temperature of the fuel-air mixture, assuming that it is a diatomic gas with a $\gamma = 1.40$. We use the adiabatic gas law

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma} \Longrightarrow p_2 = \left(\frac{V_1}{V_2}\right)^{\gamma} p_1 = (15)^{1.40} (1.01 \times 10^5) = 44.8 \times 10^5$$
 Pa

This result is 44 atmospheres, which tells you why diesel engines are so heavy to be able to withstand such high pressures.

We can work out the final temperature from $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$, where $T_1 = 300$ K. From this equation we will get $T_2 = 886K$ K or $T_2 = 613^{\circ}$ C. The temperature almost triples in absolute terms.

How much work is done by the gas in this compression if $V_1 = 10^{-3} m^3 (= 1 L)$? We have for the work done in an adiabatic change

$$W_{1\to 2} = \frac{1}{\gamma - 1} \left(p_1 V_1 - p_2 V_2 \right) = -494$$
 Joules

Work is being done on the gas during any compression. It is the angular momentum of the rotating drive shaft in the car which is doing this work. After the piston is compressed, the high temperature will self-ignite the fuel-air mixture; there is no spark plug in a diesel engine. That will deliver more power to turn the drive shaft and eventually the wheels.