REVIEW: (Chapter 4) Newton's Three Laws of Motion

First Law: The Law of Inertia

An object at rest will remain at rest unless and until acted upon by an external force.

An object moving at constant velocity will continue to move at constant velocity unless and until acted upon by an external force.
These two statements are the same when one considers that whether or not an object has velocity depends upon one's frame of reference.

Second Law: Relation between Force and Acceleration

An object will experience an acceleration **a** *in direct proportion to the force* **F** *exerted on the object. The proportionality constant is the object's mass* **m**.

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} \tag{4.7}$$

If there is no force then there must be no acceleration. If there is no acceleration then there must be no force. If there is force then there must be acceleration. If there is acceleration then there must be a force.

Third Law: Action and Reaction Forces

Every (action) force which exists has an equal an opposite (reaction) force, but the action-reaction forces NEVER act on the same body.

A force $\vec{\mathbf{F}}_{\mathbf{AB}}$ is exerted on body A by body B. By Newton's Third Law there must also be a force $\vec{\mathbf{F}}_{\mathbf{BA}}$ which is exerted on body B by body A.

$$\vec{\mathbf{F}}_{\mathbf{AB}} = -\vec{\mathbf{F}}_{\mathbf{BA}} \tag{4.11}$$

"It takes two to tango" could be just another way of stating Newton's Third Law. In other words, all forces are the result of two bodies (particles) acting on one another.

Insight into Newton's First Law of Motion

First Law: The Law of Inertia

An object at rest will remain at rest unless and until acted upon by an external force.
An object moving at constant velocity will continue to move at constant velocity unless and until acted upon by an external force.

This may be the most difficult law of all to grasp. In fact, for at least 2,000 years, until the time of Galileo and Newton, the law was simply not recognized by humankind.

The View of Aristotle

The view which pervaded human thought until the 1600s was that objects were "naturally" at rest on the surface of the Earth. In order to keep an object moving, some force was necessary, although the word "force" would not have been used. Once the force was taken away, then the object would slow down and return to its "natural" state of rest.
Of course, there were certain objects such as the Sun, the Moon, the stars and the planets, which appeared to be in a perpetual state of motion, unaided by external force. Since this motion was not "natural", then the objects had to be "supernatural" or gods.

The View of Newton (Galileo and others of the time)

The present scientific view of motion is called *Newtonian Mechanics*, after Sir Issac Newton. It was Newton who discerned the connection between motion and force, and he was able to dispel the notion of a "natural" state of rest. His First Law was truly a revolution, and we can best appreciate it through the air track demonstration shown in class.

Astronomy and Newton's First Law of Motion

We conclude the study of Newton's First Law by returning to the observations of the seemingly perpetual motion of astronomical objects such as the Sun, the Moon, and the planets.
Here again is the First Law:
An object at rest will remain at rest unless and until acted upon by an external force.
An object moving at constant velocity will continue to move at constant velocity unless and until acted upon by an external force.

Now we ask these questions relative to the First Law, where we assume that the motion is described according to a coordinate system which is fixed at the center of the Sun:

1) Is the motion of the planets consistent with the First Law?

2) Specifically, are all of these objects moving with constant velocity, and how does one define constant velocity?

3) If a given astronomical object such as the Moon is not moving with constant velocity, what inference can we derive from the First Law

With the answer to this last question, we (as was Newton) are drawn into the discussion of the Second Law.

CHAPTER 4: Newton's Second Law of Motion

The fundamental equation of mechanics is Newton's Second Law of Motion:

$$\mathbf{F} = m\mathbf{a} \tag{4.7}$$

A **FORCE** acting on an object with mass m will produce an acceleration $\mathbf{a}$. Note that this is a vector equation, so it actually represents three separate equations for the X, Y, and Z components of the force and the acceleration.

More than one force acting on an object

If there is more than one force acting on an object, the left hand side of the Newton's second law is simply replaced by the vector sum of the forces acting on the body:

$$\Sigma_{i=1}^{N}\mathbf{F}_i = m\mathbf{a} \tag{4.8a}$$

$$\Longrightarrow \Sigma_{i=1}^{N}F_{i_x} = ma_x \qquad \Sigma_{i=1}^{N}F_{i_y} = ma_y \qquad \Sigma_{i=1}^{N}F_{i_z} = ma_z \tag{4.8b}$$

It is very important that you realize that this equation applies to the *forces acting on the object*. The most difficult thing for most students is to recognize all, and only all, the forces which act on a particular object. Sometimes students will leave out certain forces, and other times students will include forces which are not acting on the object in question. The secret is to focus your attention on the object in question.

ACCELERATED MOTION

Accelerated motion occurs when the left hand side of Eq. 4.7 is not zero. Then the object *must* have an acceleration. Many of the problems you will have to work out will involved computing the acceleration of an object when the object experiences certain forces. Typical cases will include the force of gravity (weight), the *normal forces* exerted by supporting walls and floors, and the *static* or *kinetic* frictional forces.

Applications of Newton's Laws of Motion

While the three Newton's Laws of Motion look extremely simple at first sight (and really are deep down) , there are a wide variety of problems to which they can be applied. At first glance, the solutions to those problems may seem difficult for you. However, with practice in getting to think about how to analyze these problems, their solutions should come easier to you.
There are two types of problems

1) **Static Equilibrium** where there are one or more objects subjected to forces but no object has an acceleration, and

2) **Accelerated Motion** where there are one or more objects subjected to forces, and there is an acceleration.

Both of these problems are dealt with in Chapter 4 and Chapter 5.

Static Equilibrium
This is the case of Newton's second law where there is no *net* force on an object. Typically the object is at rest in the Earth's frame of reference.

$$\Sigma \mathbf{F}_i = 0$$

$$\Longrightarrow \Sigma F_{ix} = 0 \quad \text{and} \quad \Sigma F_{iy} = 0$$

$\Longrightarrow$ the sum of the Forces acting Right = the sum of the Forces acting Left
$\Longrightarrow$ the sum of the Forces acting Up = the sum of the forces acting Down
In static equilibrium problems an object will have a specified mass m. In turn, any object with a mass m at the Earth's surface will have a weight force $w = mg$, pointing downward. If the mass m is on some horizontal surface, and it has no vertical acceleration, then there must be a counter-acting force acting against the weight force. This force, acting perpendicular to the surface and **on the object**, is called the **Normal Force** of the surface, with the symbol N.

Steps in Solving Statics Problems using **The "Free Body" Approach**
Generally you will be confronted with one or more objects on which are exerted certain forces. Usually there will be the weight forces, and the objects will exert forces on one another. You will be asked to solve for one or more unknown forces. You should then follow a procedure call the **Free Body** Approach, explained on the next page.

The Free Body Approach to STATIC (EQUILIBRIUM) Mechanics Problems

1) Isolate the object(s) of interest.

2) Identify and draw all the forces acting on the object(s). This is called a *free body* diagram.

3) If there is more than one object, make sure you understand which forces are acting on which objects.

4) For each object, resolve the forces into their rectangular components.

5) Apply the Newton's Second Law equations

$$\Sigma F_{ix} = 0$$

and

$$\Sigma F_{iy} = 0$$

to the separate object(s).

6) Solve for the unknown force(s).

The Free Body Approach to NON-EQUILIBRIUM Mechanics Problems

A non-equilibrium problem has accelerated motion. One can also do a free-body approach. The only difference is in step 5) above. For the mass m of each free body, we will have Newton's Second Law as

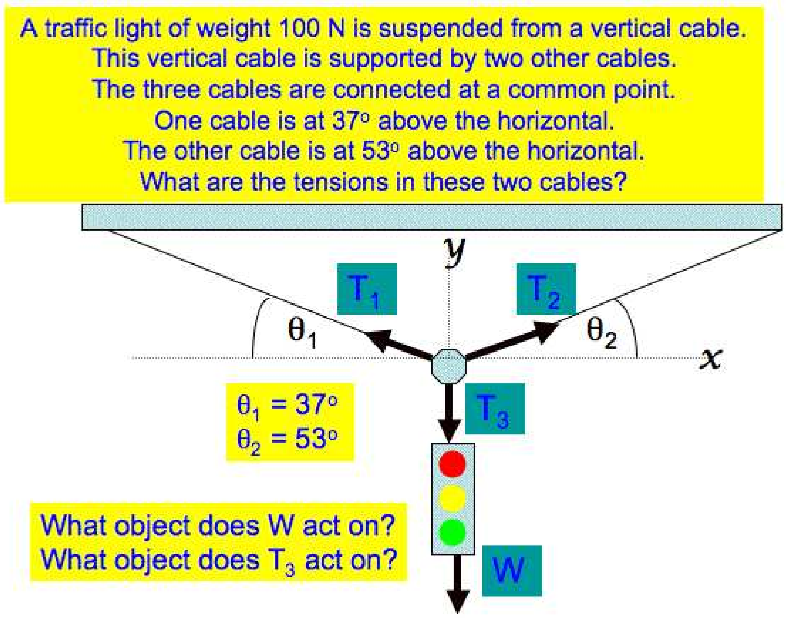
$$\Sigma F_{ix} = ma_x$$

and

$$\Sigma F_{iy} = ma_y$$

Worked Example in STATIC EQUILIBRIUM

A traffic light weighing 100 N hangs from a cable which in turn is tied to two other cables fastened to an overhead support. The upper two cables make angles of $37^{\rm O}$ and $53^{\rm O}$ with the horizontal. Find the *tensions* in the three cables.



This is a static equilibrium problem because there are no accelerations. All the objects are at rest.

We focus in on a single object, the common point (octagon in the diagram) where all three cables are connected.

We next draw an (x, y) coordinate system whose origin is at that common point. Then, we have to decide what are the directions (angles) which the three tension forces make in that coordinate system, according to our diagram.

Worked Example in STATIC EQUILIBRIUM

A traffic light weighing 100 N hangs from a cable which in turn is tied to two other cables fastened to an overhead support. The upper two cables make angles of $37^{\rm O}$ and $53^{\rm O}$ with the horizontal. Find the *tensions* in the three cables.

Solution

First construct the diagram of the physical set–up, and then isolate the objects of interest. First there is the traffic light, and second there is the *knot* where the three cables are connected. (These are knotty problems; there will often be a *knot* which is at rest while subject to three or more tensions.)

From the free-body diagram we immediately deduce the magnitude of $T_3 = 100$ N. Then we can proceed to the free body diagram of the knot. We know that the vector sum $\mathbf{T_1} + \mathbf{T_2} + \mathbf{T_3} = \mathbf{0}$, so we can write down the usual table for the analytic addition of vectors:

Force	Magnitude (N)	Angle (deg.)	X Comp. (N)	Y Comp. (N)
$\mathbf{T_1}$	T_1	143	$T_1 \cos 143^{\rm O}$	$T_1 \sin 143^{\rm O}$
$\mathbf{T_2}$	T_2	53	$T_2 \cos 53^{\rm O}$	$T_2 \sin 53^{\rm O}$
$\mathbf{T_3}$	100	270	0.00	-100.0

$$\Sigma F_x = 0 \Longrightarrow T_1 \cos 143^{\rm O} + T_2 \cos 53^{\rm O} = 0$$

$$\Sigma F_y = 0 \Longrightarrow T_1 \sin 143^{\rm O} + T_2 \sin 53^{\rm O} - 100 = 0$$

We have two equations in two unknowns (T_1 and T_2). From the first:

$$T_2 = T_1 \Big(\frac{-\cos 143^{\rm O}}{\cos 53^{\rm O}} \Big) = 1.33 T_1$$

Substitute this in the second equation to obtain

$$T_1 \sin 143^{\rm O} + 1.33 T_1 \sin 53^{\rm O} - 100 = 0$$

$$\Longrightarrow T_1 = 60.2 \textbf{ N, and } T_2 = 79.9 \textbf{ N}$$

The above solution is checked in a spreadsheet file on the class web site.

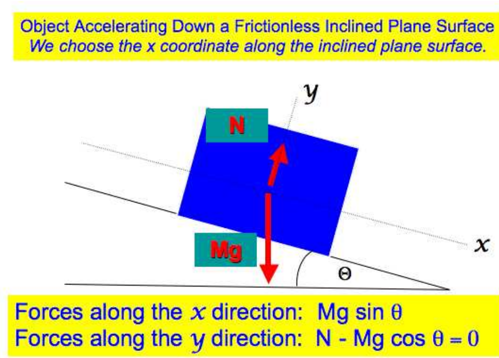
Accelerated Motion $\mathbf{F} = m\mathbf{a}$

A extremely common example of the accelerated motion type is a force acting on an inclined plane. A sled of mass m is placed on a *frictionless* hill (inclined plane) which is at an angle Θ with the horizontal. Find the acceleration of the mass down the hill.

The forces *acting on the block* are:

1) The downward weight force $\mathbf{W} = m\mathbf{g}$ exerted by gravity.

2) The normal force $\mathbf{N}$ exerted by the inclined plane, and perpendicular ("normal") to its surface. You don't know the magnitude of the normal force initially, only its direction.



In this example, and in all inclined plane problems, it is convenient to have the y axis perpendicular to the plane, and the x axis parallel to the plane. (Why is that *convenient* ?). In that coordinate system the net component of the force in the y direction is 0. How can you be sure of that? The two separate equations in 4.7 become:

$$\Sigma F_{xi} = mg\sin\theta = ma_x$$

$$\Sigma F_{yi} = N - mg\cos\theta = 0$$

There are two unknowns in these two equations: a_x, and N, and each of these unknowns can be solved for immediately. From the first equation

$$a_x = g\sin\theta$$

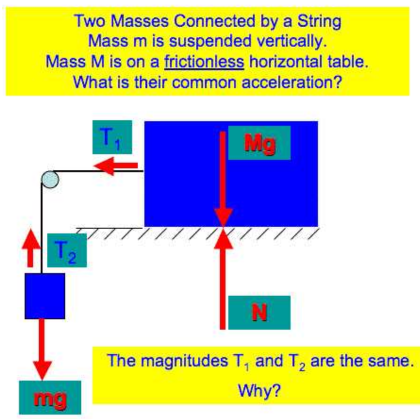
The acceleration down the plane *is independent* of the size of the mass m. All objects slide down frictionless inclined planes at the same acceleration which depends only on the size of the plane's inclination angle.

From the second equation you can obtain the magnitude of the normal force

$$N = mg\cos\theta$$

Newton's Second Law F = ma and Connected Objects

An object of mass M is on a frictionless horizontal table. This object is connected by a massless string going over a pulley onto a second object m which is falling because of gravity ? What is the acceleration of M, and what are the net forces acting on each of M and m.



It is necessary to draw a *separate free body diagram for each mass*. Then add up all the forces acting on each mass separately, and use Newton's second law in each case. For mass M the only unbalanced force acting is the tension T in the connecting string. So by Newton's second law for M becomes

$$F_M = T = Ma$$

For mass m there are two forces acting: the upward tension T and the downward weight force mg. So the Newton's second law equation for m becomes

$$F_m = mg - T = ma$$

Notice that the magnitude of the acceleration, on the right hand side of each equation, is the same in both cases. This is because the masses are connected by an inextensible string. There are two unknowns in these two equations: the magnitudes T and a. Since there are two equations, then it is possible to solve for these two unknowns. The simplest approach is just to add the two equations in order to eliminate the T unknown. You should obtain after adding

$$mg = Ma + ma \Longrightarrow a = \frac{m}{M+m}g$$

$$\Longrightarrow T = Ma = \frac{Mm}{M+m}g$$

Newton's Second Law and Static Friction

Static friction is a **Force** parallel to the surface of contact between two objects, which acts on an object to *prevent* motion from occurring. Because static friction prevents the motion from occurring, the object remains stationary. The magnitude of the static friction will then be equal to the magnitude of the external force trying to making the motion occur (what about the direction of the static friction ?). So as the external force is increasing, the static friction force must also increase. But this cannot continue indefinitely. *There is a maximum value that the static friction can have for a given object on a given surface.* That maximum is given by the equation

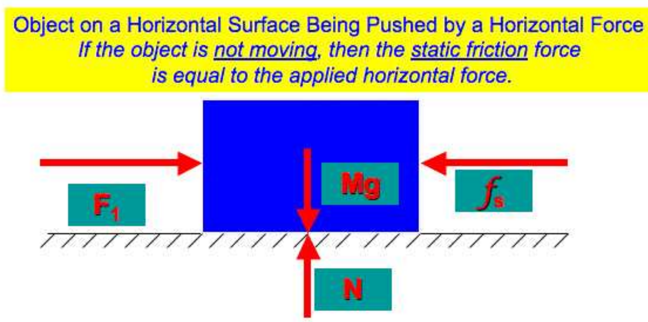
$$f_s^{max} = \mu_s \cdot N$$

where μ_s is the *coefficient of static friction*, and N is the normal force of the surface against the object.

Use of the Normal Force instead of the Weight Force
Why can't we just simply say that N is equal to the weight of the object ?
It could be that the Normal Force is less than the Weight Force, $N < W$, if there is another force supporting part of the object and $N > W$ is there is another force pressing down on the object. Also the normal force may even be horizontal instead of vertical, or at any angle $0 \leq \theta \leq 90^{\mathrm{O}}$. (Example of a block on a blackboard !)

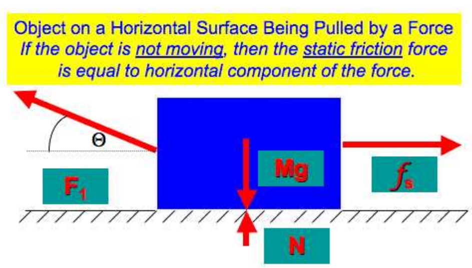
Static Friction

Case 1 A force $\mathbf{F_1}$ pushes horizontally against an object of mass M on a rough, horizontal surface. The object remains stationary. What is the maximum force of static friction ? Below is the free-body diagram up until the point at which $f_s = f_s^{max} = \mu_s N$. Until that point $f_s = F_1$ and $f_s < f_s^{max}$.



$$f_s^{max} = \mu_s \cdot N \qquad N = Mg \qquad \Longrightarrow \qquad f_s^{max} = \mu_s Mg$$

Case 2 A force $\mathbf{F_1}$ pulls at an angle Θ against a stationary object of mass M which is on a rough, horizontal surface. What is the maximum force F_1 which can be applied?



$$f_s^{max} = \mu_s \cdot N \qquad N \neq Mg$$

The normal force of the surface, in this case, is not equal to the total weight vector. The reason is that part of the weight of the object is supported by the upward component of the applied force $\mathbf{F_1}$.

$$\text{Vertical Force Equality: } N + F_1 \sin\Theta = Mg \Longrightarrow N = Mg - F_1 \sin\Theta$$

$$\text{Horizontal Force Equality: } \quad f_s^{max} = F_1 \cos\Theta$$

Substitute for $f_s^{max} \Longrightarrow \mu_s N = \mu_s(Mg - F_1 \sin\Theta) = F_1 \cos\Theta \Longrightarrow F_1 = \dfrac{\mu_s Mg}{\cos\Theta + \sin\Theta}$

Newton's Second Law and Kinetic Friction

$$\mathbf{F} = \sum_{i=1}^{N} \mathbf{F}_i = m\mathbf{a}$$

Kinetic Friction

Kinetic Friction is the **Force** of friction which "takes over" once the object starts moving. The force of kinetic friction is given by a similar equation

$$f_k = \mu_k \cdot N$$

Here we see that the kinetic friction force is always the same for a given normal force. No matter how fast the object is moving, and no matter whether it is accelerating or just moving at constant velocity, we still have the same force of kinetic friction *opposing* the direction of the motion. Since motion is actually occurring, this must mean that there is another force at least as big as, or bigger than, the force of friction.

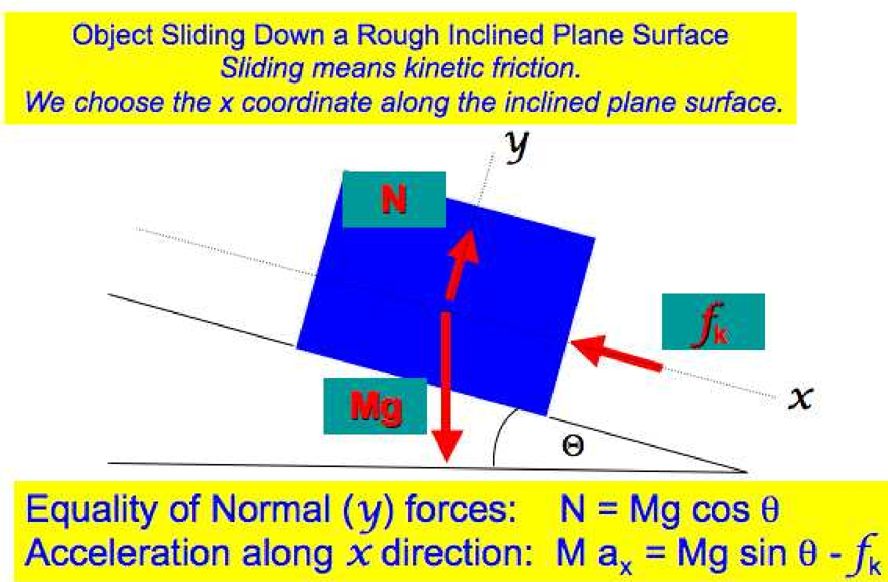


Figure 1: A 3 kg block *starting from rest* slides down a rough inclined plane with $\theta = 30^{\text{O}}$, covering a distance of 2 meters along the plane in 1.5 seconds.

Worked Problem with KINETIC FRICTION

A 3 kg block *starts from rest* at the top of a 30^{O} inclined plane slides down a distance of 2 m down the incline in 1.5 seconds. Find a) the acceleration of the block, b) the speed of the block after it has slid 2 m , c) the frictional force acting on the block, and d) the coefficient of kinetic friction between the block and the plane.

First draw the figure and indicate all the forces acting on the block. The solution to parts a) and b) can be found immediately using the kinematics equations

$$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \qquad \text{and} \qquad v_x(t) = v_{0x} + a_x t$$

As usual in inclined plane problems, the x axis is down the plane, and the y axis is perpendicular to the plane. In this case $x_0 = 0$, $v_{0x} = 0$, and we are given that the block has moved 2 m after $t = 1.5$ seconds

$$x(t = 1.5) = 2 = \frac{1}{2}a_x(1.5)^2 \Longrightarrow a_x = \frac{4}{2.25} = 1.78 \text{ m/s}^2$$

$$v_x(t = 1.5) = v_{0x} + a_x(1.5) = 2.67 \text{ m/s}$$

For part c) apply Newton's second law of motion in the x direction

$$x \text{ direction:} \qquad F_x = Mg\sin\theta - f_k = Ma_x \qquad \text{(the same } a_x \text{ as before)}$$

$$\Longrightarrow f_k = Mg\sin\theta - Ma_x = 3\cdot 9.8\sin 30^{\text{O}} - 3\cdot 1.78 = 9.37 \text{ N}$$

For part d), to obtain the kinetic coefficient of friction, apply Newton's second law in the y direction where there is no acceleration and therefore no net force:

$$y \text{ direction:} \qquad F_y = N - Mg\cos\theta = 0 \qquad \text{(because } a_y = 0\text{)}$$

Substitute for the kinetic friction force $f_k = \mu_k N = \mu_k Mg\cos\theta$ which gives

$$\mu_k = \frac{f_k}{Mg\cos\theta} = \frac{9.37}{3\cdot 9.8\cos 30^{\text{O}}} = 0.365$$