REVIEW: (Chapter 4) Newton's Three Laws of Motion

First Law: The Law of Inertia

An object at rest will remain at rest unless and until acted upon by an external force.

An object moving at constant velocity will continue to move at constant velocity unless and until acted upon by an external force.
These two statements are the same when one considers that whether or not an object has velocity depends upon one's frame of reference.

Second Law: Relation between Force and Acceleration

An object will experience an acceleration **a** *in direct proportion to the force* **F** *exerted on the object. The proportionality constant is the object's mass* **m**.

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} \tag{4.7}$$

If there is no force then there must be no acceleration. If there is no acceleration then there must be no force. If there is force then there must be acceleration. If there is acceleration then there must be a force.

Third Law: Action and Reaction Forces

Every (action) force which exists has an equal an opposite (reaction) force, but the action-reaction forces NEVER act on the same body.

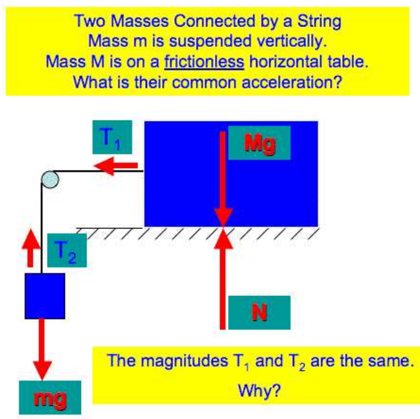
A force $\vec{\mathbf{F}}_{\mathbf{AB}}$ is exerted on body A by body B. By Newton's Third Law there must also be a force $\vec{\mathbf{F}}_{\mathbf{BA}}$ which is exerted on body B by body A.

$$\vec{\mathbf{F}}_{\mathbf{AB}} = -\vec{\mathbf{F}}_{\mathbf{BA}} \tag{4.11}$$

"It takes two to tango" could be just another way of stating Newton's Third Law. In other words, all forces are the result of two bodies (particles) acting on one another.

REVIEW: Newton's Second Law F = ma and Connected Objects

An object of mass M is on a frictionless horizontal table. This object is connected by a massless string going over a pulley onto a second object m which is falling because of gravity ? What is the acceleration of M, and what are the net forces acting on each of M and m.



It is necessary to draw a *separate free body diagram for each mass*. Then add up all the forces acting on each mass separately, and use Newton's second law in each case. For mass M the only unbalanced force acting is the tension T in the connecting string. So by Newton's second law for M becomes

$$F_M = T = Ma$$

For mass m there are two forces acting: the upward tension T and the downward weight force mg. So the Newton's second law equation for m becomes

$$F_m = mg - T = ma$$

Notice that the magnitude of the acceleration, on the right hand side of each equation, is the same in both cases. This is because the masses are connected by an inextensible string. There are two unknowns in these two equations: the magnitudes T and a. Since there are two equations, then it is possible to solve for these two unknowns. The simplest approach is just to add the two equations in order to eliminate the T unknown. You should obtain after adding

$$mg = Ma + ma \Longrightarrow a = \frac{m}{M+m}g$$

$$\Longrightarrow T = Ma = \frac{Mm}{M+m}g$$

Newton's Second Law and Static Friction

Static friction is a **Force** parallel to the surface of contact between two objects, which acts on an object to *prevent* motion from occurring. Because static friction prevents the motion from occurring, the object remains stationary. The magnitude of the static friction will then be equal to the magnitude of the external force trying to making the motion occur (what about the direction of the static friction ?). So as the external force is increasing, the static friction force must also increase. But this cannot continue indefinitely. *There is a maximum value that the static friction can have for a given object on a given surface.* That maximum is given by the equation

$$f_s^{max} = \mu_s \cdot N \qquad 5.6$$

where μ_s is the *coefficient of static friction*, and N is the normal force of the surface against the object.

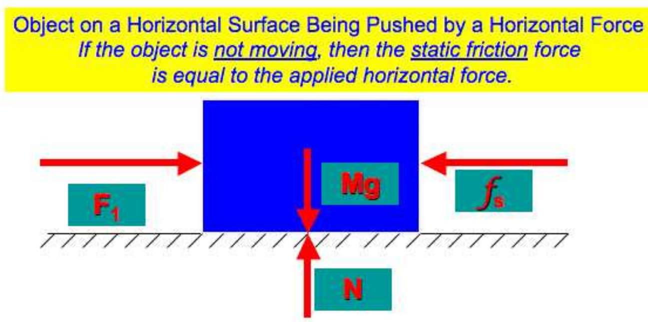
Use of the Normal Force instead of the Weight Force

Why can't we just simply say that N is equal to the weight of the object ?

It could be that the Normal Force is less than the Weight Force, $N < W$, if there is another force supporting part of the object and $N > W$ is there is another force pressing down on the object. Also the normal force may even be horizontal instead of vertical, or at any angle $0 \leq \theta \leq 90^{\rm O}$. (Example of a block on a blackboard !)

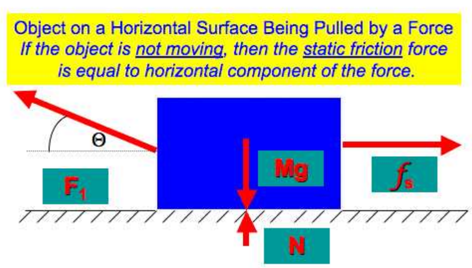
Static Friction

Case 1 A force $\mathbf{F_1}$ pushes horizontally against an object of mass M on a rough, horizontal surface. The object remains stationary. What is the maximum force of static friction ? Below is the free-body diagram up until the point at which $f_s = f_s^{max} = \mu_s N$. Until that point $f_s = F_1$ and $f_s < f_s^{max}$.



$$f_s^{max} = \mu_s \cdot N \qquad N = Mg \qquad \Longrightarrow \qquad f_s^{max} = \mu_s Mg$$

Case 2 A force $\mathbf{F_1}$ pulls at an angle Θ against a stationary object of mass M which is on a rough, horizontal surface. What is the maximum force F_1 which can be applied?



$$f_s^{max} = \mu_s \cdot N \qquad N \neq Mg$$

The normal force of the surface, in this case, is not equal to the total weight vector. The reason is that part of the weight of the object is supported by the upward component of the applied force $\mathbf{F_1}$.

$$\text{Vertical Force Equality: } N + F_1 \sin\Theta = Mg \Longrightarrow N = Mg - F_1 \sin\Theta$$

$$\text{Horizontal Force Equality: } \qquad f_s^{max} = F_1 \cos\Theta$$

Substitute for $f_s^{max} \Longrightarrow \mu_s N = \mu_s(Mg - F_1 \sin\Theta) = F_1 \cos\Theta \Longrightarrow F_1 = \dfrac{\mu_s Mg}{\cos\Theta + \sin\Theta}$

Newton's Second Law and Kinetic Friction

$$\mathbf{F} = \sum_{i=1}^{N} \mathbf{F}_i = m\mathbf{a}$$

Kinetic Friction

Kinetic Friction is the **Force** of friction which "takes over" once the object starts moving. The force of kinetic friction is given by a similar equation

$$f_k = \mu_k \cdot N \qquad 5.5$$

Here we see that the kinetic friction force is always the same for a given normal force. No matter how fast the object is moving, and no matter whether it is accelerating or just moving at constant velocity, we still have the same force of kinetic friction *opposing* the direction of the motion. Since motion is actually occurring, this must mean that there is another force at least as big as, or bigger than, the force of friction.

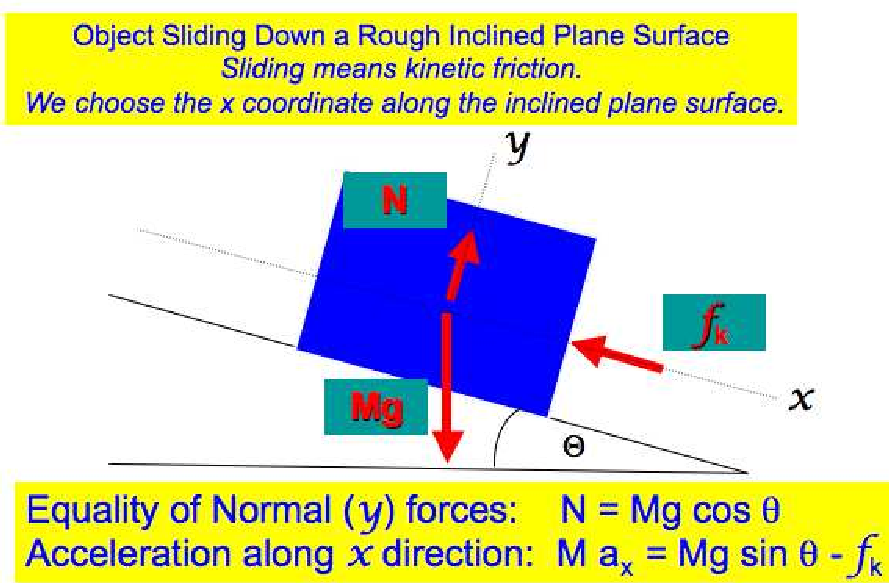


Figure 1: A 3 kg block *starting from rest* slides down a rough inclined plane with $\theta = 30^{\mathrm{O}}$, covering a distance of 2 meters along the plane in 1.5 seconds.

Worked Problem with KINETIC FRICTION

A 3 kg block *starts from rest* at the top of a 30^{O} inclined plane slides down a distance of 2 m down the incline in 1.5 seconds. Find a) the acceleration of the block, b) the speed of the block after it has slid 2 m , c) the frictional force acting on the block, and d) the coefficient of kinetic friction between the block and the plane.

First draw the figure and indicate all the forces acting on the block. The solution to parts a) and b) can be found immediately using the kinematics equations

$$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \qquad \text{and} \qquad v_x(t) = v_{0x} + a_x t$$

As usual in inclined plane problems, the x axis is down the plane, and the y axis is perpendicular to the plane. In this case $x_0 = 0$, $v_{0x} = 0$, and we are given that the block has moved 2 m after $t = 1.5$ seconds

$$x(t = 1.5) = 2 = \frac{1}{2}a_x(1.5)^2 \Longrightarrow a_x = \frac{4}{2.25} = 1.78 \text{ m/s}^2$$

$$v_x(t = 1.5) = v_{0x} + a_x(1.5) = 2.67 \text{ m/s}$$

For part c) apply Newton's second law of motion in the x direction

$$x \text{ direction:} \qquad F_x = Mg\sin\theta - f_k = Ma_x \qquad \text{(the same } a_x \text{ as before)}$$

$$\Longrightarrow f_k = Mg\sin\theta - Ma_x = 3 \cdot 9.8\sin 30^{\mathrm{O}} - 3 \cdot 1.78 = 9.37 \text{ N}$$

For part d), to obtain the kinetic coefficient of friction, apply Newton's second law in the y direction where there is no acceleration and therefore no net force:

$$y \text{ direction:} \qquad F_y = N - Mg\cos\theta = 0 \qquad \text{(because } a_y = 0\text{)}$$

Substitute for the kinetic friction force $f_k = \mu_k N = \mu_k Mg\cos\theta$ which gives

$$\mu_k = \frac{f_k}{Mg\cos\theta} = \frac{9.37}{3 \cdot 9.8\cos 30^{\mathrm{O}}} = 0.365$$

Centripetal Force and Circular Motion

Standard scientific literacy question: A object is attached to a string and the object is traveling in a horizontal circle. The string breaks. What happens to the object ? Does it go it a curved horizontal path from the point at which it broke free ?
Answer: — **absolutely not**. If the object breaks free, its horizontal path is a **straight line**. The only way for the object to go in a curved or circular path in the horizontal plane is to have a force acting perpendicular to its velocity in the horizontal plane.

The force which makes an object go in a circle is called the **centripetal force**. Centripetal force is required to produce centripetal acceleration. You cannot have one without the other. If you see circular motion you can be sure that there is a centripetal force being produced somewhere, somehow.

$$F_r = ma_r = \frac{mv^2}{r}$$

Centripetal Force Worked Example

A 1500 kg car is moving on a horizontal road which has a curve (part of a circle) with a radius of curvature of 35 m. The coefficient of *static friction* between the tires and the road is 0.50. What is the maximum speed at which the car can make this curve?
It is important that you realize that there must be a centripetal force exerted on the car (actually on the tires) by the road's surface in this case. It is this force which changes the direction of the car's velocity vector. Now the maximum static friction force is given by

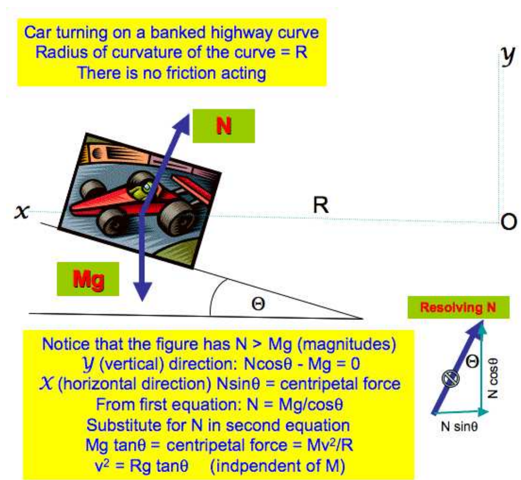
$$f_s^{max} = \mu_s N = \mu_s M g = \frac{M v_{max}^2}{R}$$

$$\Longrightarrow v_{max} = \sqrt{\mu_s g R} = \sqrt{0.5 \cdot 9.8 \cdot 35} = 13.1 \text{ m/s} \qquad \text{(independent of mass!)}$$

Do you understand why static friction is being used, and not kinetic friction?
Answer: Believe or not, the contact point of the tire on the road surface is *instantaneously at rest* if the car is not skidding. We learn about that when we discuss rotational motion.

Centripetal Force and Improved Highway Engineering

In the previous example, the centripetal force which enabled the car to make the curve came just from the force of static friction. If the force of static friction is reduced, because of wet or icy roads, then the maximum safe speed for making the turn may be drastically reduced.
In order to rely less on friction, highway engineers came up with the idea of *banking* the curves. The road surface, on a curve, becomes effectively an inclined plane. The normal reaction force of the road surface is no longer purely vertical, but it acquires a horizontal component. That horizontal component functions as the centripetal force which allows the car to turn in a circle.



In *banked* curves, it is possible for friction to be completely absent and still the car can make the turn. The centripetal force is given by the horizontal component of the normal force

$$F_{centripetal} = N \sin\theta = \frac{Mv^2}{R} \quad \text{and} \quad N\cos\theta = Mg$$

$$F_{centripetal} = \frac{Mg}{\cos\theta}\sin\theta = \frac{Mv^2}{R} \Longrightarrow v = \sqrt{Rg\tan\theta} \qquad \text{(independent of M!)}$$

Note again that the maximum speed here is independent of the mass of the car and depends just on the radius R, the banking angle θ, and the gravitational acceleration constant g

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For home study: suppose in addition to a banked angle, there was also a coefficient of static friction μ_s. How would you solve for the maximum speed in that case ?

Worked Example using CENTRIPETAL FORCE

Another example of centripetal motion is the *conical pendulum*

An object of mass m is suspended from a string, and the object moves at constant speed in a horizontal circle of radius R. The string makes an angle θ with respect to the vertical direction. What is the speed of m (in terms of θ, g, and R) ?

The key in circular motion problems is to find the force component which is acting centripetally. This force component must be directed at the center of the circle and perpendicular to the instantaneous velocity vector of the object. In this case there are two forces: 1) the weight vector acting vertically down, and 2) the tension vector of the string acting at some angle θ with respect to the horizontal. Next, analyze the acceleration of the object. In which direction(s) x (horizontal) or y (vertical) does the particle have an acceleration ? If the acceleration is 0, then the net force acting in that direction must also be 0.

For the conical pendulum, the object has no acceleration in the vertical direction. Therefore, the net force in the y direction is 0:

$$F_y = 0 = T_y - W = T\cos\theta - Mg \Longrightarrow T\cos\theta = Mg$$

In the horizontal (x) direction, *there is an acceleration !* This is the centripetal acceleration because the horizontal velocity is changing continuously. Therefore, there is a net force in the horizontal direction

$$F_x = T_x = Ma_x \Longrightarrow T\sin\theta = \frac{Mv^2}{R}$$

Take these two equations and divide the second by the first:

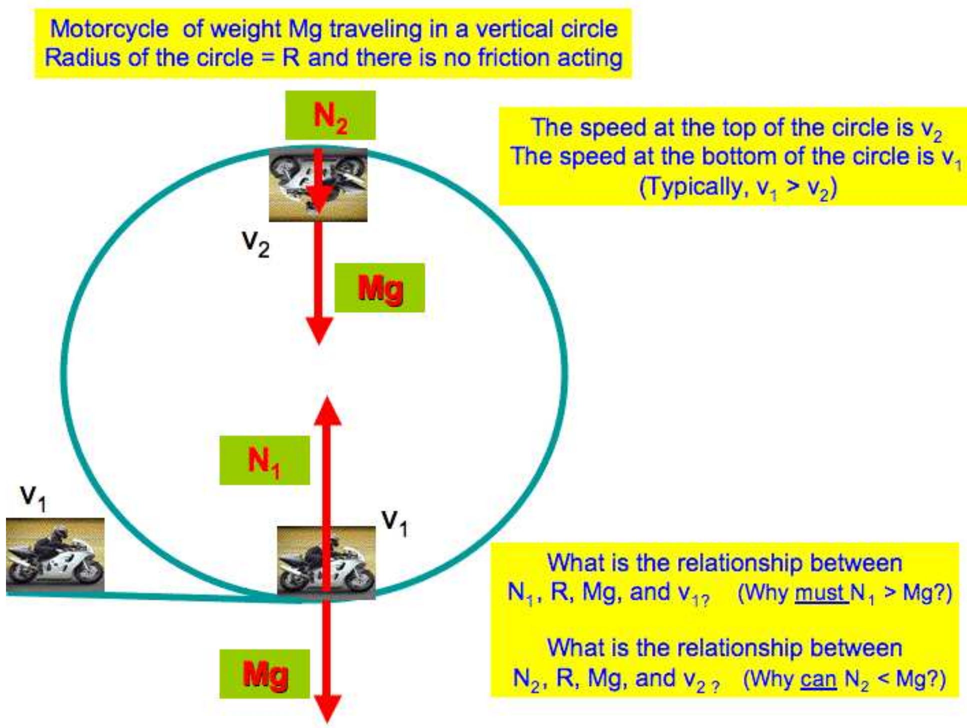
$$\frac{T\sin\theta}{T\cos\theta} = \frac{Mv^2}{RMg}$$

$$\tan\theta = \frac{v^2}{Rg} \Longrightarrow v = \sqrt{Rg\tan\theta} \qquad \text{(independent of M!)}$$

Notice that the conical pendulum result is the same as the banked highway result. The only difference in the two problems is that a normal force N is changed into a tension force T. The free-body diagrams are otherwise identical.

Centripetal FORCE in a VERTICAL Circle

So far all these examples of centripetal force have been for motion in a horizontal circle. The last example for this chapter will be for motion in a *vertical circle*. A motorcycle stuntman (Allo "Dare Devil" Diavolo was the first to do this trick in 1901) rides a motorcycle around a "loop-the-loop" track. What speed must he have had at the top of the loop (where he is upside-down) in order not to fall off the loop, where the loop has a radius R? (Friction is not a factor.)



Since he is moving in a circle, he must have a centripetal force. At the top of the loop, that force is pointing straight down. The question you must ask yourself is where is that force coming from. The answer depends on how fast he is moving. In general, if his speed at the top of the loop is v, then there **must** be a centripetal force F_c given by:

$$F_c = \frac{Mv^2}{R}$$

Now, as to where this force is coming from, the easiest answer is gravity. There will **always** be a gravitational or weight force $w = Mg$.

Suppose that the weight force is bigger than the centripetal force. In other words suppose that the speed v is not so great. In that case, only part of the weight force w is "used up" by the centripetal force. So the motorcycle will fall off the track.

Centripetal FORCE in a VERTICAL Circle

$$F_c = \frac{Mv^2}{R}$$

$$w = Mg$$

Suppose instead that the weight force is less than the centripetal force. Then the weight force will be insufficient to provide enough centripetal force. Where does the rest of the centripetal force come from? It comes from the track, specifically the **Normal Force** of the track on the motorcycle.

The limiting case is for the weight force to be exactly equal to the centripetal force. In that case, there is no need for a normal force but the motorcycle does not fall off the track.

$$F_c = w \quad \text{and} \quad \frac{Mv^2}{R} = Mg$$

$$\Longrightarrow v = \sqrt{gR}$$

Note that the minimum speed v does NOT depend on the mass M. So it does not depend on how heavy the man is, nor how heavy the motorcycle is. It only depends upon g and the radius R.