

1. Vector  $\vec{A} = (5.7, -3.6)$  and  $\vec{B} = (-9.8, -6.5)$ . Then the difference  $\vec{D} = \vec{B} - \vec{A} = (-15.5, -2.9)$ . So the magnitude  $D = \sqrt{(-15.5)^2 + (-2.9)^2} = 15.8$ .

**Answer = C (16)** 72% correctly answered

2. Rotating a vector changes its direction. One cannot add a scalar to a vector, although one can multiply a vector by a scalar. Only vectors of the same magnitude can be summed to zero (they would be anti-parallel). A vector with a non-zero component must have a magnitude of at least the size of that component. The magnitude of a vector is independent of the coordinate system used to describe the components of the vector. That is why having vectors in Physics is so useful, since Physics does not depend upon the choice of any (inertial only) coordinate system.

**Answer = A** 69% answered correctly

3. The vector  $\vec{A} = (3.9, -4.0)$ . The angle of the vector is  $\tan^{-1}(A_x, A_y) = \tan^{-1}(3.9, -4.0) = -45.6^\circ = 314.3^\circ$ . Note that if the problem had specified  $\vec{A} = (-3.9, 4.0)$ , and you divided first with your calculator, your calculator might have returned  $-45.6^\circ$  as well. However, the correct answer would have been in the second quadrant instead of the fourth quadrant.

**Answer = B (314)** 80% answered correctly

4. The vector  $\vec{A}$  has a magnitude of 4.8 and is in the second quadrant, at an angle of 18 degrees with respect to the  $y$  axis. Its  $x$  and  $y$  components are  $(A \cos(18 + 90), A \sin(18 + 90)) = (-1.48, 4.57)$ , where the  $90^\circ$  is being added because the angle was specified with respect to the  $y$  axis instead of the  $x$  axis.

**Answer = C (-1.5, +4.6)** 82% answered correctly

5. The net displacement  $\vec{AC}$  is  $-5$  meters, and this displacement is accomplished in 28 seconds. Thus the average velocity  $\vec{v}_{AC} = \vec{AC}/28 \text{ seconds} = -0.179 \text{ m/s}$ . The negative sign indicates that the direction of the average velocity is the negative  $x$  direction.

**Answer = A (-0.179 m/s)** 74% answered correctly

6. The total distance traveled (from A to B to C) is  $5 + 10 + 10 = 25$  meters, in 28 seconds. So the average speed is  $25/28 = 0.893 \text{ m/s}$ .

**Answer = D (0.893 m/s)** 72% answered correctly

7. An object can have momentarily zero velocity but non-zero acceleration, such as a ball thrown vertically upwards at the highest point of its motion. A changing velocity must mean a non-zero acceleration, which could be constant or varying in time. An object could be moving East with an East acceleration, meaning that the speed of the object is increasing. Conversely, an object moving East with a West acceleration is decelerating, meaning its speed is decreasing. An object with a constant non-zero velocity cannot have any acceleration, either constant or changing.

**Answer = D** 76% answered correctly

8. This is a one-dimensional motion problem with constant acceleration  $a_y = -g = -9.8 \text{ m/s}^2$ . We have  $y_0 = 70$ , with a choice of origin at ground level. Also,  $v_0 = +31.9 \text{ m/s}$ . To find out when the ball hits the ground we use the position kinematic equation with constant acceleration:

$$y(t) = y_0 + v_0 t - \frac{1}{2}gt^2$$

and solve for the time  $t$  value such that  $y(t) = -70$  meters. This results in a quadratic equation for which the positive (physically appropriate) solution is 8.2 seconds.

**Answer = B (8.2 s)** 91% answered correctly

9. The particle starts out with a positive velocity (positive slope in the  $x(t)$  graph), but that velocity diminishes monotonically with time. At the latest time shown, the slope is almost horizontal, meaning almost zero speed. Only the velocity graph shown in choice A depicts these characteristics.

**Answer = A** 83% answered correctly

10. A golf ball hit at  $60^\circ$  angle above the horizontal is a projectile meaning that it has zero horizontal acceleration and  $-9.8 \text{ m/s}^2$  constant vertical acceleration. Its speed is never zero, although its vertical component of velocity is zero at its highest point.

**Answer = B (acceleration is always  $9.8 \text{ m/s}^2$  downward)** 90% answered correctly

11. The stone has a horizontal component of velocity  $v_{0x}$  as  $25.0 \times \sin(40) = 16.07$  m/s. The sine of the angle is being used because the angle was given with respect to the vertical direction. Similarly the vertical component of velocity  $v_{0y}$  is  $-25 \times \cos(40) = -19.15$  m/s, where the negative sign is obviously needed for a stone be thrown in the downward direction. To get the height of the cliff  $y_0$  we use the position kinematic equation, since we are told that the stone takes 3.75 seconds to reach the ground ( $y = 0$ ).

$$y(3.75) = 0 = y_0 + v_{0y}t - \frac{1}{2}gt^2 \implies y_0 = 4.9(3.75)^2 + (19.15)(3.75) = 140.7 \text{ meters}$$

**Answer = D (141 m)** 70% answered correctly

12. For the horizontal distance traveled we simply multiply the horizontal component of the initial velocity by the time of travel:  $16.07 \times 3.75 = 60.3$  meters.

**Answer = D (60.3 m)** 77% answered correctly

13. Each scale will read 10 kg, just as all points of a massless string contain the same tension force if the string is supporting a mass in the vertical direction.

**Answer = D** 73% answered correctly

14. We must first calculate the mass of the object, and then calculate the weight of this mass with the Moon's gravitational acceleration value. The mass of the object is the force divided by the resulting acceleration  $m = F/a = 10/8.6 = 1.16$  kg. So the weight on the Moon is  $1.16 \times 1.62 = 1.88$  Newtons.

**Answer = A (1.9 N)** 95% answered correctly

15. This is a Newton's third law problem. The wall reacts to the push of the man by pushing against the man with the same magnitude of force that the man has. Answer C is always correct. Answer D *could* sometimes be correct, if the man was standing up *and* pushing against the wall in a direction to the left. On the other hand, if the man was in a chair rigidly attached to the floor, there may not be any force on his feet no matter what direction he is pushing. If the man was standing on the floor, it would be the force of static friction from the floor to the man's shoes which would be transmitted up to the man's hands. The man would not be able to push as strongly on a slippery floor. (The man might have been sitting on the floor, pushing with his feet instead.)

**Answer = C** 98% answered correctly

16. The pull force  $P$  must support the weight of the two masses, which is 125 N. The tension force  $T$  needs to support only the 100 N weight. Hence  $P = 125 = 100 + 25 = T + 25$  N.

**Answer = E (P = T + 25 N)** 70% answered correctly

*Part I: mean score was 12.70 +/- 2.35 correct out of 16*

*79% mean score, with 92 students*

Part II.

1. a) What initial speed  $v_0$  does Lula-Belle have?

We use the projectile trajectory equation  $y(x)$  since we know the  $(x, y)$  values at Billy-Joe's position, and we also know the initial angle  $\theta_0$ . We can choose the origin to be at Lula-Belle's initial position. So we have

$$y(x) = x \tan \theta_0 - \frac{g}{2v_0^2 \cos^2 \theta_0} x^2 \implies$$

$$v_0^2 = \frac{gx^2}{2 \cos^2 \theta_0 (x \tan \theta_0 - y)} = \frac{9.8(8.2)^2}{(2 \cos^2(53))(8.2 \tan(53) - 6.1)} (\text{m/s})^2 = 190.2 (\text{m/s})^2$$

$$v_0 = 13.8 \text{ m/s}$$

Part a) is worth 9 points. If you knew that you should use the trajectory equation, but you had a math error in solving for  $v_0$ , you would get 7 points. If you wrote down the wrong trajectory equation, you would get 6 points. You could also try to solve this part by writing simultaneous  $x(t)$  and  $y(t)$  equations, and then substituting for  $t$  in the  $y(t)$  equation. This would effectively be the trajectory equation approach, if you did it all correctly. If you used  $v_y^2 = v_{0y}^2 - 2gy$  and assumed  $v_y = 0$  at Billy-Joe's position ( $y = 6.1$  m), that would be wrong. You would lose 5 points for that incorrect assumption.

1. b) What is the velocity of Lula-Belle at Billy-Joe's position?

For a projectile the horizontal velocity component  $v_0 \cos \theta_0 = 13.8 \cos 53 = 8.30$  m/s is constant. The vertical velocity component can be obtained by now using  $v_y^2 = v_{0y}^2 - 2gy$ , since we know  $v_{0y} = 13.8 \sin 53 = 11.0$  m/s, and  $y = 6.1$  m. Thus we get  $v_y = 1.33$  m/s. Notice that this  $v_y$  component is non-zero and still positive. Therefore, Lula-Belle is still on her way up at Billy-Joe's position. She has not yet reached her maximum height. The velocity magnitude is computed as  $\sqrt{(8.30)^2 + (1.33)^2} = 8.41$  m/s, and the direction is  $\tan^{-1}(1.33/8.30) = 9.1^\circ$ .

Part b) is worth 4 points. If you started from a wrong  $v_0$  value but otherwise had the solution done correctly, you would still get 4 points.

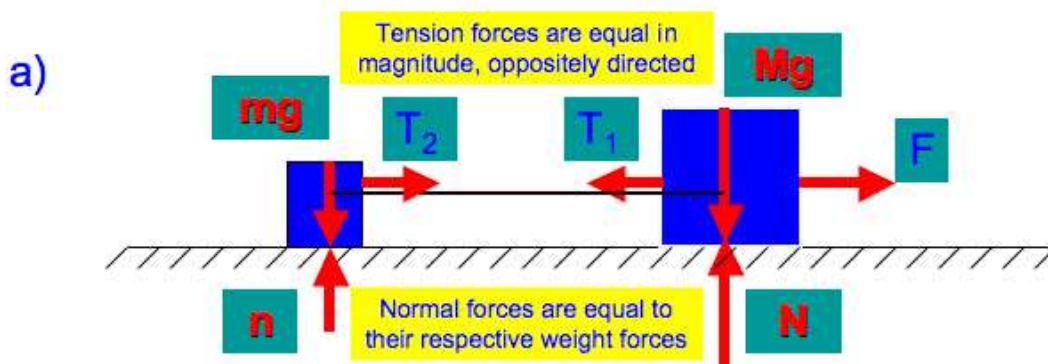
1. c) To find the horizontal distance traveled  $x$ , you would again use the trajectory equation with  $y = -8.6$  meters, and  $v_0 = 13.8$  m/s. This would result in a quadratic formula whose solution is  $x = 23.8$  meters.

Part c) is worth 5 points. As before, if you had the right equation, but the wrong  $v_0$  value while everything else was correct, you would still receive 5 points. If you had a math error in your quadratic solution, you would lose 2 points.

## Problem 2

Two Masses (3 kg, 1.5 kg) Connected by a String on a Horizontal Frictionless Surface  
Horizontal Force  $F$  Pulls on the Larger Mass Producing an Acceleration

- a) Draw the Free Body Diagrams, b) What is the Magnitude of  $F$  ?  
c) What is the Tension in the String ?



- b) The force  $F$  accelerates both  $M$  and  $m$  with the same acceleration, given by Newton's second law  

$$F = (M + m) a = (3.0 + 1.5) \text{ [kg]} 2.50 \text{ [m/s}^2\text{]} = 11.25 \text{ N}$$

- c) The tension  $T$  in the string is the magnitude of  $T_1$  or  $T_2$   
 We can apply Newton's second law to the smaller mass  

$$T = m a = 1.5 \text{ [kg]} 2.50 \text{ [m/s}^2\text{]} = 3.75 \text{ N}$$

The net force on  $M$  is  $F - T = 11.25 - 3.75 = 7.5 \text{ N}$ , and  

$$Ma = 3.0 \text{ [kg]} 2.50 \text{ [m/s}^2\text{]} = 7.5 \text{ N}$$
 which matches

Part a) is worth 9 points. Part b) is worth 4 points. Part c) is worth 5 points. If you did not draw in the vertical forces in the free body diagrams, you lost 4 points. If you did not clearly show that the tension forces on the two masses were equal in magnitude but opposite in direction, then you lost 3 points. If you added in friction forces, you lost 3 points.