1. First calculate how much total energy is added to the 2000 kg of sand in 60 seconds. The power will be that total energy divided by 60 seconds. The total energy is the potential energy increase in rising 12 meters plus the kinetic energy in acquiring a speed of 5 m/s.

$$E_{\text{total}} = Mgh + \frac{1}{2}Mv^2 = 2000(9.8 \times 12 + 0.5 \times (5)^2) = 2.602 \times 10^5 \text{Joules}$$

$$P = \frac{E}{t} = \frac{2.602 \times 10^5 \text{Joules}}{60 \text{ seconds}} = 4340 \text{ Watts}$$

Answer is A

2. A ball which is rising is still being affected by the force of gravity, which is external to the ball. So its momentum is not conserved. In fact, its momentum is continually decreasing since its speed is continually decreasing as it rises. Its potential energy and kinetic energy are also continuously changing, but the mechanical energy sum remains constant.

Answer is A

3. If the same force acts over twice the distance then the kinetic energy change will be twice as much. Since $K \equiv mv^2/2$, a doubling of the kinetic energy means that the speed has increased by $\sqrt{2}$.

Answer is D

4. First we need to get the (x, y) coordinates of each mass

1) 2 kg mass at $(2, 0)$
2) 6 kg mass at $(5, 2)$
3) 4 kg mass at $(3, 5)$
4) 8 kg mass at $(0, 3)$

Now we can compute x_{cm} and y_{cm} according to the sums over positions weighted by the masses:

$$x_{\text{cm}} = \frac{\Sigma_{i=1}^4 m_i x_i}{\Sigma_{i=1}^4 m_i} = \frac{2*2 + 6*5 + 4*3 + 8*0}{2 + 6 + 4 + 8} = 2.3$$

$$y_{\text{cm}} = \frac{\Sigma_{i=1}^4 m_i y_i}{\Sigma_{i=1}^4 m_i} = \frac{2*0 + 6*2 + 4*5 + 8*3}{2 + 6 + 4 + 8} = 2.8$$

Answer is A

5. This problem is solved in three steps. When the hockey puck leaves the top of the cliff, it is traveling horizontally with a speed v_1. It will then be a projectile falling to the ground from an initial height $y_0 = 8.5$ meters and with vertical acceleration g. This projectile will also travel at constant horizontal speed v_1. We can find v_1 by computing the fall time t, and then divided the horizontal distance traveled (6.2 meters) by that time t. Then we can use conservation of energy to find the energy at the top of the cliff, which must be all kinetic energy when the puck started with speed v_0.

$$\text{Falling body with no initial speed } y(t) = 0 = y_0 - \frac{1}{2}gt^2$$

$$\text{Time to fall 8.5 meters } \Longrightarrow t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{17}{9.8}} = 1.32 \text{ seconds}$$

$$\text{Horizontal speed at the top of the cliff } v_1 = \frac{6.50}{1.32} = 4.84 \text{ m/s}$$

$$\text{Total Energy at the top of the cliff } = mgh + \frac{1}{2}mv_1^2$$

$$\text{Total Energy at the start } = \frac{1}{2}mv_0^2$$

$$\text{Conservation of energy } = \frac{1}{2}mv_0^2 = mgh + \frac{1}{2}mv_1^2$$

$$\Longrightarrow v_0^2 = 2gh + v_1^2 = 2 * 9.8 * 8.5 + (4.84)^2 \text{ (m/s)}^2 \Longrightarrow v_0 = 13.8 \text{ m/s}$$

Answer is D

6. Conservation of energy requires that at the highest point of the motion, where the speeds and the kinetic energies are zero, then the potential energies must equal the initial kinetic energies. The initial potential energy can be taken as zero. Ball B with twice the speed of ball A then must travel four times as high as ball A since it has four times the kinetic energy of ball A, and potential energy goes linearly with the height mgh.
Answer is B

7. Simple plug-in solution from the banked curve speed equation
$v_{\text{max}} = \sqrt{Rg\tan\theta} \Longrightarrow \theta = \tan^{-1}(v_{\text{max}}^2/Rg) = \tan^{-1}((78)^2/(190 * 9.8))$
$\theta = 73.0^{\text{o}}$.
Answer = B

8. This is a static equilibrium situation where the net forces on each of the two objects A and B are both zero. For object A that means the tension in the connecting rope is equal to the weight of A. That tension $T = w_A$ acts to the right on object B. The force Q acts to the left on B, attempting to move B to the left. That means there is a static friction force f_s^{max} which is at its maximum acting to the right on B, preventing B from moving to the left. In turn, the static friction force is given by the normal force of the table, which is the sum of the weight of B and the downward push force P which is acting vertically on B.

$$f_s^{\text{max}} = \mu_s N = \mu_s(w_B + P)$$

$$Q = T + f_s^{\text{max}} = w_A + \mu_s N = w_A + \mu_s(w_B + P)$$

$$Q = 19 * 9.8 + 0.40 * (15 * 9.8 + 60) = 269 \text{ N}$$

Answer = C

9. In a completely inelastic collision, where the colliding masses move together at a common final velocity, only the momentum is conserved. Some fraction of the kinetic energy is lost to heat and other forms.

Answer = D

10. In all four cases, the stones have the same initial total energy. They are at the same height and at the same speed with the same mass. Therefore, at the bottom of the bridge they still must have the same total energy. Taking the water surface under the bridge to be the zero of potential energy, that means that the stones all have the same kinetic energy when they hit the water. Hence they have the same speeds. (Their times of travel, however, are different in each case.)

Answer = E

11. The initial kinetic energy of the mass is partially converted into potential energy in the spring while the rest of the kinetic energy is lost to friction. The initial kinetic energy of the mass is $mv^2/2 = 2.5(1.2)^2/2 = 1.8$ Joules. Since -0.50 Joules is lost to friction, and the mass stops, that means there is $+1.3$ Joules stored as potential energy in the spring. The spring has been compressed 0.05 meters. So

$$U^{\text{spring}} = \frac{1}{2}kx^2 = \frac{1}{2}k(0.05)^2 = 1.3 \Longrightarrow k = \frac{2 * 1.3}{(0.05)^2} = 1040 \text{ N/m}$$

Answer = E

12. The knot juncture where all three ropes are connected must be in equilibrium. In particular the tension force acting to the right (=55 N) must be equal to the tension force acting to the left (= $63 \cos\theta$ N). Hence $\theta = \cos^{-1}(55/63) = 29^{\text{O}}$.
Answer = C

13. The force component of gravity acting down the plane is $mg \sin\theta$. This force component acts over a displacement $s = 1.6$ meters down the plane. Therefore the work done by gravity is $mg \sin\theta(s) = 8.0 * 9.8 \sin(40^{\text{O}}) * (1.6) = +81$ Joules.
Answer = D

14. The potential energy of the ball is continuously decreasing because its height is continuously decreasing. Therefore its kinetic energy, and thus its speed, are continuously increasing. As for its acceleration, the acceleration for an inclined plane of angle θ above the horizontal is $g \sin\theta$. This hill has a continuous decrease of inclination angle (tangent or slope), and so the acceleration tangent to the hill is also decreasing.
Answer = B

15. Doubling the compression of a spring means that it contains four times the amount of potential energy. When all of that potential energy is transferred to the mass, the mass will have four times the kinetic energy. Quadrupling the kinetic energy means that the speed has doubled.
Answer = C

16. Since the kinetic energy is a positive mass times the square of a number (which must be positive), then kinetic energy is always positive. The potential energy can be negative, depending on where the zero level of potential energy is defined. If the potential energy is negative enough and larger than the kinetic energy, then the total energy will be negative.
Answer = B

17. The average force is found from the impulse equation using the net momentum change $\overline{F}\Delta t = \Delta p = m\Delta v$. In this case, taking the positive direction as the initial direction of the ball, $\Delta v = 11 - (-11) = 22$ m/s. So $\overline{F} = (0.8) * (22)/0.05 = 352$ N.
Answer = C

18. As usual, we take the x direction to be along the hill (inclined plane), and the y direction to be perpendicular to the hill. The forces acting parallel to the hill are the $T\cos(30^{\rm O})$ component of the tension in the positive x direction, and in the negative x direction the kinetic friction f_k and the weight component $mg\sin(10^{\rm O})$. In the direction normal to the hill there is the N normal force acting in the positive y direction, the $T\sin(30^{\rm O})$ component of the tension also in the positive direction, and the weight component $mg\cos(10^{\rm O})$ in the negative direction.

$$\text{Perpendicular to the hill, zero net force } T\sin(30^{\rm O}) + N - mg\cos(10^{\rm O}) = 0$$

$$\Longrightarrow N = mg\cos(10^{\rm O}) - T\sin(30^{\rm O})$$

$$\text{Parallel to the hill, non-zero net force } T\cos(30^{\rm O}) - f - mg\sin(10^{\rm O}) = ma$$

$$T\cos(30^{\rm O}) - \mu N - mg\sin(10^{\rm O}) = ma$$

$$T\cos(30^{\rm O}) - \mu(mg\cos(10^{\rm O}) - T\sin(30^{\rm O})) - mg\sin(10^{\rm O}) = ma$$

$$T = \frac{ma + \mu mg\cos(10^{\rm O}) + mg\sin(10^{\rm O})}{\cos(30^{\rm O}) + \mu\sin(30^{\rm O})}$$

Since the acceleration is given as $a = 0.4g$ then

$$T = \frac{60 * 0.4 * 9.8 + 0.1 * 60 * 9.8\cos(10^{\rm O}) + 60 * 9.8 * \sin(10^{\rm O})}{\cos(30^{\rm O}) + 0.1 * \sin(30^{\rm O})}$$

$$T = 431 \text{ N}$$

Answer = D: add 6 points if correct answer, lose 3 points if incorrect answer

19. The objects start out with zero kinetic energy and only potential energy. The object with mass $2m$ has twice the potential energy of the object with mass m at the same initial height. At the bottom of the building, where the potential energy can be taken as 0, the kinetic energy of the $2m$ mass must be twice that of the m mass.
Answer = B

20. For an elastic collision, we have two equations. One is the conservation of momentum, and the second is the relative velocity of approach is the negative of the relative velocity of separation.

$$\text{Conservation of momentum } m\vec{v}_i = m\vec{v}_f + M\vec{V}$$

$$\text{Relative velocity of approach and separation } \ \vec{v}_i = -(\vec{v}_f - \vec{V})$$

$$\Longrightarrow \vec{V} = \vec{v}_i + \vec{v}_f$$

$$\Longrightarrow \vec{V} = 2.9\,\hat{\mathbf{i}} - 0.9\,\hat{\mathbf{i}} = 2.0\,\hat{\mathbf{i}}$$

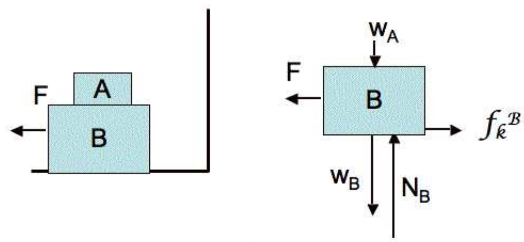
where we take the initial direction of m to be the positive x direction. Substituting this result for $\vec{V}$ into the conservation of momentum equation, we will get

$$4.2 * 2.9\,\hat{\mathbf{i}} = -4.2 * 0.9\,\hat{\mathbf{i}} + M * 2.0\,\hat{\mathbf{i}}$$

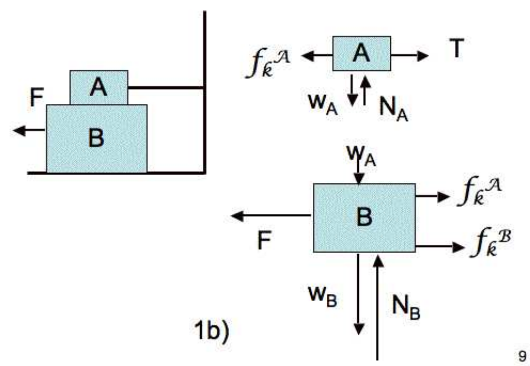
$$M = 4.2 * (2.9 + 0.9)/2.0 = 8.0 \text{ kg}$$

Answer = B

1a. In the figure the mass A weighs 1.2 N, and the mass B weighs 3.60 N. The coefficient of kinetic friction between all the surfaces is 0.300 . What is the magnitude of the horizontal force F needed to move blocks A and B together with the same constant speed (Hint: Since A is at rest with respect to B, there is no friction force between A and B when A and B are moving together at constant speed.) (12 points) *This is problem 5.67, page 174, in the text.*



1a)



1b)

Solution 1a

For block A only the gravity force w_A and the normal force N_A from B act on A. The normal force from B has magnitude w_A. There are no friction forces acting on A from B since A and B are at rest with respect to each other. No force is needed to keep A moving at constant velocity. The free-body diagram for A in this part is omitted since it only tells us that $N_A = w_A$.

For block B the free-body diagram is shown. The normal force from the table acting on B is N_B, and the magnitude $N_B = w_A + w_B$. Since B is moving to the left with respect to the table because of force F, then there is a kinetic friction force f_k^B acting to the right on B from the table opposing that motion:

$$f_k^B = \mu_k N_B = 0.3 * (1.2 + 3.6) = 1.44 \text{ N}$$

Since B is not being accelerated, then the applied force F is just equal to this kinetic friction force: $F = 1.44$ N.

1b. Block A is now fixed by a cord attached to the wall on the right. What is the magnitude of the horizontal force F needed to move block B at constant speed, while block A remains at rest with respect to the table but not with respect to B (8 points)?

Solution 1b

In this case, because there is motion of B underneath A, then there is a friction force acting to the left on A coming from B, and symbolized by f_k^A. This friction force to the left on A is balanced out by the tension force T pulling on A to the right. The magnitude of this friction force from B acting on A is

$$f_k^A = 0.3 * N_A = 0.3 * 1.2 = 0.36 \text{ N}$$

The net force on A is still zero.

For block B the forces acting are the same as in the previous part, except that there is another friction force from A acting to the right on B. This is a Newton's Third Law reaction force: B acts on A, and A reacts by acting with the same magnitude force on B.

The total friction force on B acting to the left becomes

$$f_k = f_k^A + f_k^B = 1.80 \text{ N}$$

In turn, this is the amount of force F which must be applied to the left in order for B to move at constant velocity to the left.

Summarizing, in this second part, there are two friction forces acting on B. The first comes from the table, as in the first part, and the second comes from block A because in the second part A and B are in motion relative to each other.

2a. A mass of 0.3 kg is on a frictionless, horizontal table. The mass is connected (fixed) to a massless spring which has a force constant of 3.00 N/m. The mass is pushed against the spring such that the spring is compressed to be 10 cm shorter than its natural length, at which point the mass is released. What is the speed of the mass when the spring is 5 cm longer than its natural length (12 points)?

Solution 2a

The solution uses conservation of energy. The initial potential energy of the spring, when compressed by 0.10 meters, is changed into kinetic energy of the mass and a smaller potential energy of the spring when stretched to 0.05 meters.

Initial mechanical energy $E_i = U_i^{\text{spring}} = \frac{1}{2}kx_i^2 = \frac{1}{2} * 3.0 * (0.10)^2 = 1.5 \times 10^{-2}$ J

$$\text{Final mechanical energy } E_f = U_f^{\text{spring}} + K_f = \frac{1}{2}kx_f^2 + \frac{1}{2}mv_f^2$$

$$E_f = \frac{1}{2} * 3.0 * (0.05)^2 + \frac{1}{2} * 0.3 * v_f^2$$

Since energy is being conserved $E_f = E_i = 1.5 \times 10^{-2}$ J, which leads to

$$v_f^2 = \frac{2 * 1.5 \times 10^{-2} - 3.0 * (0.05)^2}{0.3} = 7.5 \times 10^{-2} (\text{m/s})^2$$

$$v_f = 2.74 \times 10^{-1} \text{ m/s}$$

2b. Suppose that there is constant kinetic friction acting between the mass and the table in the part 2a) above, over the 15 cm horizontal distance that the mass travels. How much kinetic friction force would be necessary such that the mass has zero speed when it reaches the position above where the spring is 5 cm longer than its natural length (8 points)?

Solution 2b

If the mass has zero speed, then all the kinetic energy that it had in part 2a is being dissipated by frictional work (heating). Since we know the distance over which the frictional force is acting, we can compute this work by the usual force times distance formula. Treating everything as a positive quantity

$$f_k s = \frac{1}{2}mv_f^2 \Longrightarrow f_k = \frac{mv_f^2}{2s}$$

$$f_k = \frac{0.3 * (0.274)^2}{2 * 0.15} = 7.51 \times 10^{-2} \text{ N}$$