1. Free credit. I decided that this problem was too tricky, since people might assume that the wheel would slow down and stop. You can work out a solution where the wheel goes through 0 angular velocity and starts rotation in the opposite direction. It will eventually (after 79 seconds) achieve an angular velocity which is $-\sqrt{2}\omega_i$, meaning that it has double its initial energy.

2. The spinning ice-skater effect was demonstrated and explained in class with the *professor-holding-dumbbells-while-seated-in-spinning-chair* demonstration.
Answer = D, conservation of angular momentum

3. We can solve this problem by using either conservation total energy, or by conservation of angular momentum, between the minimum distance r_1 and the maximum distance r_2. The angular momentum method is easier

$$L_1 = L_2 = mv_1r_1 = L_2 = mv_2r_2 \Longrightarrow v_1 = \frac{r_2}{r_1}v_2$$

The speed v_2 at the maximum distance is given as 908 m/s. This gives the maximum speed at the closest distance as $v_1 = 5.46 \times 10^4$ m/s.
Answer C

4. This problem says that a steel wire of length 0.50 m and radius 0.00075 m stretches by 0.0011 m when a weight w is suspended with this wire. According to the Young's Modulus definition

$$Y \equiv \frac{(F/A)}{\Delta L/L_0} \Longrightarrow F = YA\frac{\Delta L}{L_0} = 2.0 \times 10^{11}\pi(7.5 \times 10^{-4})^2\frac{1.1 \times 10^{-3}}{0.50} = 778 \text{ N}$$

Answer = D

5. The net torque is related to the angular acceleration by $\tau = I\alpha$ with $I = 2MR^2/5$ in this case ($M = 1.85$ kg, $R = 0.45/2$ m). We can compute the angular acceleration from $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$, where $\omega_0 = 2.40 * 2\pi$ rad/sec, and $\theta - \theta_0 = 18.2 * 2\pi$ radians. Finally, we will have

$$\tau = \frac{2}{5}(1.85)(0.225)^2\frac{(2.40 \times 2\pi)^2}{2 \times 18.2 \times 2\pi} = 0.0372 \text{ N-m}$$

Answer = A

6. Say that the disk has mass M_1 and radius R_1, while the sphere has mass M_2 and R_2. At the top of the plane, the disk has potential energy M_1gh, while the sphere has potential energy M_2gh, where h is the height of inclined plane. At the bottom of the inclined plane the potential energies are both defined to be zero, and the kinetic energies equal to the initial potential energies

$$K_1 = M_1gh = \frac{1}{2}M_1v_1^2 + \frac{1}{2}I_1\omega^2 = \frac{1}{2}M_1v_1^2 + \frac{1}{4}M_1R_1^2\frac{v_1^2}{R_1^2} = \frac{3}{4}M_1v_1^2$$

$$\Longrightarrow v_1 = \sqrt{\frac{4gh}{3}} \quad \text{speed of c.m. of disk}$$

$$K_2 = M_2gh = \frac{1}{2}M_2v_2^2 + \frac{1}{2}I_2\omega^2 = \frac{1}{2}M_2v_2^2 + \frac{1}{5}M_2R_2^2\frac{v_2^2}{R_2^2} = \frac{7}{10}M_2v_2^2$$

$$\Longrightarrow v_2 = \sqrt{\frac{10gh}{7}} \quad \text{speed of c.m. of sphere}$$

The sphere is going faster at the bottom of the inclined plane for any height h, and this does not depend on the mass nor the radius of the sphere. We did a class demonstration using a solid disk and a hoop to show the same qualitative result: the object with the smaller moment of inertia will win the race to the bottom because proportionately less kinetic energy is in rotational motion and proportionately more kinetic energy is in translational motion.
Answer D

7. Moment of Inertia I is analogous to mass m, *e.g.* linear $K.E. = mv^2/2$, and rotational $K.E. = I\omega^2/2$
Answer = A

8. We must first calculate the center-of-mass, and then calculate the moment of inertia with respect to the c.m.

$$x_{cm} = \frac{40*0 + 60*5 + 100*7}{40 + 60 + 100} = 5.0 \quad ; \quad y_{cm} = \frac{40*7 + 60*2 + 100*10}{40 + 60 + 100} = 7.0$$

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2$$

$$I = 40((0-5)^2 + (7-7)^2) + 60((5-5)^2 + (2-7)^2) + 100*((7-5)^2 + (10-7)^2)$$

$$I = 3800 \text{ kg-m}^2$$

Answer = A

9. In this problem, the dropping mass m loses potential energy mgh with $h =$ 1.5 meters. That energy goes into translational kinetic energy $K_T = mv^2/2$ for m, and rotational kinetic energy $K_R = I\omega^2/2$ for the wheel. We have

$$mgh = mv^2/2 + I\omega^2/2$$

If we can determine v, I, and ω, then we can calculate m. We will calculate the speed $v = at$ from the linear acceleration a since we know the time of fall $t = 2$ seconds. We calculate a from t since we know the distance h of the fall:

$$h = at^2/2 \Longrightarrow a = 2h/t^2 = 2(1.5)/4 = 0.75 \text{ m/s}^2$$

We now calculate $v = at = 0.75(2.0) = 1.5$ m/s, and we can calculate the angular acceleration $\alpha = a/R = 0.75/0.40 = 1.875$ rad/s^2. Knowing the value of α enables to calculate $\omega = \alpha t = 3.75$ rad/s. (As a check $\omega = v/R = 3.75$ rad/s.) Lastly, we can determine I from $\tau = I\alpha = TR \Longrightarrow I = TR/\alpha$, where T is the tension in the string operating on the wheel to produce torque τ.

$$I = 20 * 0.4/1.875 = 4.27 \text{ kg-m}^2$$

Substituting back in the conservation of energy equation produces

$$m = \frac{I\omega^2}{2gh - v^2} = \frac{(4.27)(3.75)^2}{2 * 9.8 * 1.5 - 1.5 * 1.5} = 2.2 \text{ kg}$$

Answer = E

10. A net torque of 0, say about the c.m., means that there is no angular *acceleration*, but there could be a constant angular velocity. There could also be a force through the c.m. which causes a linear acceleration without a torque.
Answer = D

11. For the first sphere the kinetic energy $K_1 = I_1\omega_1^2/2 = MR^2\omega_1^2/5$. For the second sphere of mass $M_2 = 2M$ and $R_2 = 2R$, and $\omega_2 = 2\omega$, then its rotational kinetic energy is $(2M)(2R)^2(2\omega)^2/5 = 32MR^2\omega^2/5$, which is 32 times as much.
Answer = E

12. The horizontal force from the wall is symbolized as F, the weight force of the ladder as W_L, and the weight force of the block as W_B. We can solve for F by using only the torque equation about a pivot point located where the ladder touches the ground. The horizontal distance along the ground from the wall to the pivot point is $\sqrt{5*5-3.1*3.1}=3.92$ m. (This problem is the same as the Lancelot-on-Ladder problem done in class with Lancelot at the top of the ladder.)

$$\Sigma\tau = 0 = F*3.1 - W_L*3.92/2 - W_B*3.92$$

$$F = \frac{10*9.8*1.96+80*9.8*3.92}{3.1} = 1053 \text{ N}$$

Answer = C

13. The mass M_X of planet X is not given, but it can be obtained from knowing the radius R_X and the surface acceleration g_X (see Lecture 16, pages 1–2):

$$g_X = G\frac{M_X}{R_X^2} \Longrightarrow GM_X = g_X R_X^2$$

For the weight of a mass m at a distance h above the surface of planet X to be equal to the weight on the Earth's surface we have

$$mg_E = \frac{GM_X m}{(R_X+h)^2} \Longrightarrow (R_X+h)^2 = \frac{GM_x}{g_E} = \frac{g_X R_X^2}{g_E}$$

$$h = R_X\left(\sqrt{\frac{g_X}{g_E}}-1\right) = 5.16\times10^7\left(\sqrt{\frac{13.2}{9.8}}-1\right) = 8.30\times10^6 \text{ meters}$$

Answer E

14. This answer was actually given in the MasteringPhysics Assignment 12, item 13 for the *Satellite in Orbit*. The extra hint for this item[1] emphasized that MasteringPhysics comment, and specifically mentioned the Space Shuttle astronauts appearing "weightless" was continuous falling but never reaching the Earth.

Answer E

[1] http://www.hep.vanderbilt.edu/~maguirc/Physics116SP08/physics116a_hintsAssignment12.html

15. For any spherical mass M with radius R, the escape speed from the surface is given by $v_{esc} = \sqrt{2GM/R}$. For a mass m at the surface with a vertical speed $v_1 = 0.70\sqrt{2GM/R}$, the initial total energy $K + U$ is

$$E_1 = \frac{1}{2}mv_1^2 + U(R) + \frac{1}{2}m(0.49)\frac{2GM}{R} - G\frac{Mm}{R}$$

At its maximum distance r from the center of M, when its speed becomes 0, the total energy will be only potential energy

$$E_2 = -G\frac{Mm}{r}$$

Since energy is constant, then $E_1 = E_2$ and we can solve for r

$$-G\frac{Mm}{r} = \frac{1}{2}m(0.49)\frac{2GM}{R} - G\frac{Mm}{R} \Longrightarrow r = \frac{1}{1 - 0.49}R = 2.0R$$

Answer = E

16. The first thing to notice is that the triangle formed by the 4 m cable, the 4 m length along the beam, and the 4 m along the vertical wall constitutes an equilateral triangle. This means the angles in this triangle are all $60^{\rm o}$. In turn the tension T force in the 4 m cable is making an angle of $+120^{\rm o}$ with the beam (direction of lever arm from P to the action point of T). The weight force of the ladder and the weight force of the block are at angle $-120^{\rm o}$ from the direction of the beam. The negative sign will give a negative torque (clockwise) for these two forces about the pivot point P, while the T force gives a counterclockwise torque. ($W_{\rm beam} = 100 * 9.8$ N, $W_{\rm block} = 700 * 9.8$ N, $\sin(-120) = -\sin(120)$)

$$\Sigma\tau = 0 = T * 4 * \sin(120) + W_{\rm beam} * 2 * \sin(-120) + W_{\rm block} * 6 * \sin(-120)$$

$$T = \frac{W_{\rm beam} * 2 * \sin(120) + W_{\rm block} * 6 * \sin(120)}{4 * \sin(120)} = 10,780 \text{ N}$$

Answer = C

Part II. Answer both questions, 18 points each. Partial credit will be given only if the work done is clear and correct

1. 1.50 kg grinding wheel is a solid disk ($I = MR^2/2$) with radius of 0.1 m. (This is problem 10.29 in the textbook.)

a) What constant torque will bring it from rest to an angular speed of 1200 rev/min in a time of 2.5 s? (7 points)

$$\tau = I\alpha \quad ; \quad \omega_f = \omega_0 + \alpha t, \omega_0 = 0 \Longrightarrow \alpha = \frac{\omega_f}{t} = \frac{1200 * 2\pi}{60}\frac{1}{2.5} = 50.3 \text{ rad/s}^2$$

$$\tau = \frac{1.5 * (0.1)^2}{2}(50.3) = 0.377 \text{ N-m}$$

Assuming that the wheel was initially turning in the positive (counter-clockwise) direction, this torque is also in the positive direction.

b) Through what angle in radians has it turned? (5 points)

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 = 0.0 + 0.0 + \frac{1}{2}(50.3)(2.5)^2 = 157 \text{ rad}$$

c) How much work in Joules has been done by the torque, using your answers to parts a) and b) ? (3 points)

$$W = \tau\theta = 0.377 * 157 = 59.2 \text{ Joules}$$

d) What is the final rotational kinetic energy in Joules of the wheel? (3 points)

$$K_f = \frac{1}{2}I\omega_f^2 = 59.2 \text{ Joules}$$

The final kinetic energy is the same as in work done in part c), which is an example of the work-energy theorem used in rotational motion.

Several of you mis-read the problem as the wheel starting at 1200 rev/min and then slowing down to rest. Therefore, you said the final rotational energy was zero. I took off one point if you made this mistake.

A number of you also did not convert correctly, or at all, from rev/min to radians/sec. So you did not get the correct numerical answers in the MKS system. I took off two points for this error.

2. In a region of outer space where there are no other stars or planets there are two spherical masses $M_1 = 50$ kg and $M_2 = 200$ kg, each with a radius of 0.40 meters. The masses are initially 100 meters apart, and held at rest. The masses are released. (This is one of the suggested end-of-chapter problems, 12.53, whose solutions were posted in the Oak Assignments area before the test. The numbers have doubled from the textbook version.)

a) What is the magnitude of the gravity force exerted by M_1 or M_2? (6 points)

$$F_G = G\frac{M_1 M_2}{r^2} = 6.67 \times 10^{-11}\frac{50 * 200}{(100)^2} = 6.67 \times 10^{-11} \text{ N}$$

I did not take off if you said that the separation distance was 100.8 m instead of 100.0 m.

b) As the masses accelerate towards one another, is the total linear momentum conserved? (2 points)

Yes, there are no external forces acting. The c.m. remains at rest too.

c) When the masses are 50 m apart, what is the speed of each mass? (6 points)

Evaluate the change in potential energy which will go into the sum of the kinetic energies of the two masses. The speeds v_1 and v_2 of the two masses will be in inverse proportion to their masses because the two masses will have equal, linear momentum magnitudes: $M_1 v_1 = M_2 v_2 \Longrightarrow v_2 = (M_1/M_2)v_1$.

$$\Delta U = GM_1M_2(\frac{1}{50} - \frac{1}{100}) = 6.67 \times 10^{-11}\frac{50 * 200}{100} = 6.67 \times 10^{-9} \text{ Joules}$$

$$\frac{1}{2}M_1v_1^2 + \frac{1}{2}M_2v_2^2 = \Delta U = \frac{1}{2}(M_1v_1^2 + M_2(\frac{M_1}{M_2})^2v_1^2) = \frac{1}{2}\frac{M_1(M_2 + M_1)}{M_2}v_1^2$$

$$v_1 = \sqrt{\frac{2\Delta U M_2}{M_1(M_2 + M_1)}} = 1.46 \times 10^{-5} \text{ m/s} \quad ; \quad v_2 = \frac{M_1}{M_2}v_1 = 3.65 \times 10^{-6} \text{ m/s}$$

Notice that the speeds are in the ratio 4:1. Many of you worked out solutions which had the speeds being in the ratio 2:1. I took off two points if you made such an error.

d) When the surfaces of the two masses finally touch, by how much has the center of the 50 kg mass moved from its initial position? (4 points)

We can work this problem out using the fact that the center-of-mass position stays constant. For simplicity, since the absolute value of the coordinates does not matter, take the initial position of M_1 as $r_1 = 0$ and the initial position of M_2 is then $r_2 = 100$ m. The center of mass will be at

$$r_{cm} = \frac{M_1 r_1 + M r_2}{M_1 + M_2} = \frac{200}{250} 100 = 80 \text{ m}$$

When the masses finally touch, their centers will be 0.8 meters apart

$$r'_2 - r'_1 = 0.8 \text{ m} \Longrightarrow r'_2 = r'_1 + 0.8 \text{ m}$$

The c.m. position will still be the same at 80 meters, even though the individual coordinates have changed

$$r_{cm} = \frac{M_1 r'_1 + M_2 r'_2}{M_1 + M_2} = 80 \text{ m}$$

$$r_{cm} = \frac{M_1 r'_1 + M_2 (r'_1 + 0.8)}{M_1 + M_2} = 80 \text{ m}$$

$$r'_1 = \frac{80(M_1 + M_2) - M_2(0.8)}{M_1 + M_2} = 79.4 \text{ m}$$

This 79.4 meters is the distance M_1 has moved, since it started at 0. For the second mass $r'_2 = 80.2$ meters. The second mass has moved $100 - 80.2 = 19.8$ m.

The textbook solution states that (non-constant) acceleration for M_1 will always be four times that of M_2. So the distance that M_1 moves will be four times that of M_2. From that, and knowing that the masses have moved a total of $100 - 0.8 = 99.2$ meters, you can also worked out how much each individual mass has moved. In the solution worked out above you can see the ratio $79.4/19.8 = 4$, as expected.