Special Relativity Consequences

NOTE: This Lecture is supplemented by the 30 minute video shown in class.

Special relativity of motion is an amazingly simple theory. The two basic postulates of special relativity are easily understandably, but have startling consequences.

The Two Special Relativity Postulates
#1 *All the Laws of Physics may be derived in any inertial frame. No physics measurement will distinguish any preferred reference frame.*

#2 The speed of light c is the same for all observers, independent of their own motion or the motion of the light source.

These two postulates were introduced by Albert Einstein in 1905, and were based on his attempts to reconcile the theory of Electromagnetism as developed in the 1800s with the theory of Mechanics developed two hundred years earlier. In the end it was Newton's Mechanics which had to be revised and Electromagnetism which survived unscathed.
One of the historical mysteries of science is whether Albert Einstein was aware of the results of Michaelson and Morely which essentially are the best experimental proof of these two postulates. The best evidence is that he did not know about the results, somewhat surprising because Einstein knew just about everything in the physics of his time. Instead, he based is special relativity ideas more on the papers of Lorentz who was trying also to reconcile Electromagnetism and Classical Mechanics.

Gedanken Experiments to Disprove Absolute Time
The first most startling result of the theory of special relativity is the collapse of the idea of simultaneity, or equivalently of absolute time identical for all observers whether moving or at rest relative to one another.
Einstein conjured up a lot of so-called "thought experiments" (*gedanken* experiments in German) based on the two postulates to prove this effect, since it was so difficult to do true experiments at the time. Most of these experiments had to do with moving trains very common in Europe, although now days we can think of moving space ships.

Special Relativity Consequences

No Absolute Time

The first Gedanken experiment involves a moving train in which an observer O' (Liz) is seated exactly in the center. Two lightning bolts strike each end of the car, just as the observer O' passes another observer O (Mark) who happens to be standing alongside the train tracks. The light from the two lightning bolts reaches O at exactly the same time according to how O sees things. However, O' will see the light from the front of the car arriving before the light from the rear of the car. Hence, $O\prime$ will say that the one lightning bolt preceded the other.
But you might say O' is wrong because she is in a moving frame of reference. However, O' may not know she is in a moving frame of reference, and may never know in some cases of moving reference frames. In particular, O' cannot determine that she is moving by measuring a different value for the speed of light compared to what O will measure. Moreover, from her point of view it might be O who is moving backwards and therefore he got things mixed up in his frame. Hence, O''s viewpoint of non-simultaneous events is just as valid as O's viewpoint of simultaneous events.

Time Dilation

We saw above that two observers moving relative to one another may be in disagreement with regards to when two "events" are simultaneous. An "event" means something happened at a given spatial location (x, y, z), and a given time t. This suggests something might be happening to their clocks.
To show this is true, Einstein thought up of another experiment. He used again the same moving train with the same two observers O and O'. Observer O' shines a brief flash of light from the floor of the train car up to a mirror on the ceiling from which the light is reflected down to the floor again. Somehow O' manages to get the time difference between when the flash of light leaves the floor to when it returns. (That's the nice thing about Gedanken experiments, all the really hard technical parts are left out.) He calls this time $\delta t'$. And since she knows how far it is from floor to ceiling, distance d, she can use this time interval Δt to make a clock calibration.

Time Dilation

Time intervals as measured by two observers

Liz will measure a time interval:

$$\Delta t' = \frac{2d}{c}$$

This means that she will calculate the speed of light as:

$$\Longrightarrow c = \frac{2d}{\Delta t'}$$

Meanwhile, back alongside the tracks, observer O Mark sees things a little differently. In his view, the light flash has not simply traveled straight up and down, but rather has gone on two diagonal paths of length $c\Delta t/2$ which form the hypotenueses of two right triangles. Each of the triangles has a common height d, and an equal base length $v\Delta t/2$. Note that we are using a different symbol Δt for the time interval as measured by Mark.

So Mark will use the Pythagorean theorem to calculate that:

$$(c\Delta t/2)^2 = (v\Delta t/2)^2 + d^2$$

This he will measure Δt to be

$$\Delta t = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c\sqrt{1 - v^2/c^2}}$$

Now we can relate Δt to $\Delta t'$ by substituting $\Delta t' = 2d/c$. This produces

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}}$$

The time $\Delta t'$ measured by Liz is shorter than the time Δt measured by Mark!

Now you see again the Lorentz contraction factor $1.0/\sqrt{1 - v^2/c^2}$, which occurs so often in relativity that it is given its own special symbol γ

$$\gamma = \frac{1.0}{\sqrt{1 - v^2/c^2}}$$

Time Dilation

The result of this thought experiment, based on the constancy of the speed of light, is the following:

Two observers moving relative to one another and measuring the time interval between two events will come up with different values for the time interval.

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}} = \gamma \Delta t'$$

$$\gamma = \frac{1.0}{\sqrt{1 - v^2/c^2}}$$

Again, it makes no sense to ask "Who is right?, since they both are right in their own frames of reference.

Proper Time

There is an important difference between the two observers in the previous Gedanken experiment. Liz, in the moving train car, measured the start and stop times at the same space location. So her events can be written as (x'_1, y'_1, z'_1, t'_1) and (x'_2, y'_2, z'_2, t'_2) where $x'_1 = x'_2$, $y'_1 = y'_2$, $z'_1 = z'_2$, and $t'_2 = t'_1 + \Delta t'$

On the other hand Mark, standing alongside the tracks, saw the start and stop times occur at two different space locations. Suppose we take the x direction to be the one along which the train is moving with speed v. Then Mark's events are written as (x_1, y_1, z_1, t_1) and (x_2, y_2, z_2, t_2) where $x_1 = x_2 + v\Delta t$, $y_1 = y_2$, $z_1 = z_2$, and $t_2 = t_1 + \Delta t$.

The reference frame in which the interval between two events is measured at the same place defines the *proper time* interval for the events. The *proper time* is always measured with a single clock at rest in its own rest frame.

Proof of Time Dilation

Now Gedanken experiments are fine for illustrating a point, but in reality they prove nothing physically about the validity of the special relativity assumptions. Confirmation of the time dilation prediction would come only about forty years after the publication of the theory of special relativity.

Experimental Proof of Time Dilation

Muon Decay

The best known example of time dilation is the observation of high energy particles called *muons* coming down from the upper atmosphere onto the surface. A muon is nothing more than a "heavy" electron, about 200 times the mass of an electron. It is created in the upper atmosphere when atoms there are hit by high energy protons called cosmic rays which travel all through the universe.

The big difference, besides the mass, between an electron and a muon is that a muon is unstable. It will decay after about 2.2 microseconds into an electron plus a couple of neutrinos. For all practical purposes the muon is traveling near the speed of light, say $0.99c$ when it is created. Were it not for relativistic time dilation, then the muon would decay after just about 600 meters, and never reach the Earth's surface. However, because of time dilation, the muon's internal clock runs much slower. In fact, for a speed $v = 0.99c$ we have

$$\gamma = 1.0\sqrt{1 - v^2/c^2} \approx 7.1$$

Hence, to a stationary observer looking at the moving muon, the muon's lifetime will be more like 16 microseconds. And in 16 microseconds, the muon could make it a distance of 4800 meters. That's about 15,000 feet which is where the muons could be created in abundance.

In 1976, muons were created in the CERN accelerator laboratory and accelerated to a speed of $0.9994c$. This then gave a $\gamma \approx 30$, and the accelerated muons were observed to live about 30 times as long as muons created with near 0 speed.

Use of Real Clocks

Time dilation applies to real clocks as well. This was demonstrated in 1971 when four identical cesium atomic clocks were synchronized, and then two of them were flown in an airplanes, one going West and the other East. Relative to the ground based clocks, the eastward flying clock lost 59 nanoseconds, while the westward flying clock gained 273 nanoseconds. These results, taking into account that the Earth based clocks were also moving in space, were exactly consistent with the predictions of special relativity

The Twin Paradox

Soon after Einstein's publication of the theory of special relativity and its time dilation predictions, there came a serious objection which became know as the *Twin Paradox*. This scientific riddle was the subject of many theoretical papers, until the flight of the cesium clocks which resolved the issue once and for all.

The premise of the paradox is very simple. Two twins, Speedo and Goslo, are both 20 years old. Speedo builds herself a rocket ship and accelerates to near the speed of light on a journey to another star 30 light years away from Earth. She then returns home, having aged only about 10 years because of time dilation. However, on Earth about 80 years has elapsed and her brother is looking like George Burns just before he died.

The paradox is the following: Could we look at the situation from Speedo's point of view that it was Goslo who went ("backwards") at near the speed of light. After all, all motion is relative. In that case it should be Goslo who stayed younger and Speedo who aged.

Now both conclusions cannot be correct, and when the spaceship returns to the Earth someone must be older.

The resolution of the paradox is that the situation is not completely symmetrical. We can distinguish the motion of the two twins because one twin was accelerating (and decelerating) and the other was not. So the proper frame to analyze the motion is the inertial frame, which is Goslo's frame. He is the one who ages, just has it was for the stationary cesium clocks in the airplane experiment.

Length Contraction

We have seen that there is a *proper time* interval between two events taking place at the same physical location in a given reference frame. In any other reference frame moving with respect to this one, the measured time interval will be longer by the Lorentz factor γ

There is also the concept of a *proper length* for an object. The proper length is the length of the object as measured in the reference frame where the object is at rest. The length of the object as measured in any reference frame in which the object appears to be moving will be *contracted* by the same Lorentz factor. Hence we speak of *Lorentz contraction* of moving object:

$$L = \frac{L_P}{\gamma} = (1 - \frac{v^2}{c^2})L_P$$

Here L is the length of the object as measured in a frame in which the object appears to be moving, and L_P is the length as measured in the objects own rest frame. The length contraction takes place only along the direction of the motion.

The frame of reference for the proper time measurement is always different from the frame of reference for the proper length measurement.

Chapter 9: The Lorentz Coordinate Transformation

Consequences of the Relativity Postulates

We have seen that special relativity, and especially the principle that the speed of light is the same for all persons, leads to dramatic consequences. The most important of these are the dismissal of absolute time or simultaneity for all observers.

With time no longer as an absolute, we need a new set of equations which will enable us to express mechanics quantities in any inertial reference frame moving at a speed v with respect to another frame. Previously there was the Galileo Equations:

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

where we can always assume that the relative motion is in the x direction.

Now with relativity there are a new set of equations linking the two reference frames. These equations are known as the Lorentz coordinate transformation, after H. Lorentz who was mentioned so prominently in the video as being Einstein's most important teacher. These equations, which can be proved relatively easily, are:

$$x' = \gamma(x - vt) \qquad 9.8a$$

$$y' = y \qquad 9.8b$$

$$z' = z \qquad 9.8c$$

$$t' = \gamma(t - \frac{v}{c^2}x) \qquad 9.8d$$

$$\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$$

The time transformation, Eq. 9.8d, in particular is the most strange. This equation essentially says that the time as measured by one person is dependent upon the time *and* position as measured by a person moving with speed v.

The Lorentz Velocity Transformations

Galileo Classical Velocity Transformation Equation

Again, in Newton's time, there was a very simple way to compute the speed of an object u' as observed in a frame moving with a speed v with respect to another frame in which the object's speed was measured to be u. We simply subtracted the speed of the moving frame (assumed to be in the x direction):

$$u'_x = u_x - v; \quad u'_y = u_y; \quad u'_z = u_z$$

Lorentz Relativistic Velocity Transformation

As you might expect, since Galileo's coordinate transformation equations no longer work, neither will his velocity transformation. And this has to be the case, otherwise we could get into the situation of adding speeds such that a particle moves faster than light speed. That, according to relativity, is impossible.

For simplicity we again take two reference frames moving at a speed v with respect to each other, in the x direction. We can obtain from the Lorentz coordinate transformation, the following velocity transformation equations:

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2}$$

$$u'_y = \frac{u_y}{\gamma(1 - u_x v/c^2)}$$

$$u'_z = \frac{u_z}{\gamma(1 - u_x v/c^2)}$$

Note that now even the perpendicular velocity components transform differently from the Galileo case. This is because of the time dilation.

You should prove to your self that in the limit of very small speeds v of relative motion, then the Lorentz transformations become equal to the Galileo transformations.

Example Using Lorentz Velocity Transformation Equations

Addition of Velocities

Newcomers to the theory of relativity often wonder how it can be that nothing can be made to go faster than the speed of light. For example, you might think of a person going very fast, near the speed of light, and then throwing an object at a high speed in this moving frame. Then you might say that relative to a person not moving, then the thrown object should be going faster than light speed.

However, the Lorentz velocity transformation equations simply do not allow that to happen. A good example from the text is on page 273.

A person is on a motorcycle going at a speed 0.8c relative to a stationary observer. The motorcyclist throws a ball at a speed of 0.7c relative to himself in the same direction that the motorcycle is traveling. What speed is seen by the stationary observer.

In the classical Galileo transformation, one would simply add the two velocities in the same direction. This would give a speed of $1.5c$ which is impossible according to relativity.

To see how this comes about, just use the Lorentz velocity transformation equation:

$$u_x = \frac{u'_x + v}{1 + u'_x v/c^2}$$

In this case, the speed of the moving frame $v = 0.8c$. The speed as seen in the moving frame is $u'_x = 0.7c$. So we have

$$\text{Stationary observer sees } u_x = \frac{0.8c + 0.7c}{1 + (0.7c)(0.8c)/c^2} = \frac{1.5c}{1.56} = 0.962c$$

No matter how large you make v, the relative speed, and no matter how large you make the speed in the moving frame u' (but always less than c), you will never get the speed in the stationary frame to be greater than c.

On the other hand, if you have a speed $u' = c$ in the moving frame, then you can just substitute in and find that the stationary observer will also measure $u = c$ no matter what the relative speed v between the two observers.

Other Relativistic Kinematic Variables

All the classical kinematic variables have expression in relativity of course. Those expressions must always reduce to the classical (Newton or Galileo) forms when the speed u of the object is very much less than c.

Relativistic Momentum

The expression for relativistic momentum for a particle of mass m and velocity $\vec{u}$ is simple enough

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1 - u^2/c^2}} = \gamma m\vec{u} \qquad 9.14|$$

Again, as the speed u becomes very much less than c, this means γ approaches one and the relativistic momentum and the classical momentum become equal.

Relativistic Newton's Second Law

Classically, Newton's Second Law is stated as $\vec{F} = m\vec{a}$. The relativistic version is

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad 9.16$$

However, because of the Eq. 9.8 time transformation change, and in particular the complication of the speed dependent denominator, then it is not possible to change Eq. 9.16 into Eq. 9.14.

Relativistic Kinetic Energy

Once you have the expression Eq. 9.16 for relativistic force, it is straightforward but somewhat tedious (see page 297) to calculate the work done by a force over a certain distance. That becomes also the Kinetic Energy, for which the result is:

$$K = \frac{mc^2}{\sqrt{1 - u^2/c^2}} - mc^2$$

If you expand this expression as on page 298, and drop the higher order terms (if u is very much less than c), then you will recover the classical expression $K = mv^2/2$.

The Equivalence of Mass and Energy

Relativistic Kinetic Energy and the Total Energy

The expression for relativistic kinetic energy

$$K = \frac{mc^2}{\sqrt{1 - u^2/c^2}} - mc^2$$

can be rewritten as

$$K = \gamma mc^2 - mc^2$$

Einstein stared at this expression for a while and then decided to call mc^2 the *rest mass energy.* It was a profound insight. No one before had equated mass and energy. They seemed to be fundamentally distinct quantities. But Einstein saw that this classical concept also had to fail. Mass was actually just another form of energy.

Once he had the rest mass energy concept, Einstein then simply said that an object moving at a speed u has a total energy E given by

$$E = \frac{mc^2}{\sqrt{1 - u^2/c^2}} = \gamma mc^2$$

The quantity m was an *invariant*, the same in all frames of reference.

Fusion and Fission

This chapter concludes with a discussion of fusion and fission. Very simply these two processes are based on the $E = mc^2$ formula. In fusion, one starts with two light masses, say two deuterium nuclei, and then fuses (combines) these two into another nucleus called Helium. The Helium nucleus has less mass than the two deuterium nuclei. So energy is released according to the mass difference. On the other hand, in fission, a heavy nucleus splits into two lighter nuclei. Again the final state has less total mass than the initial state, so energy is released.

Naturally, you might wonder has to why the masses of nuclei behave the way they do. It's a pretty simple explanation involving a balance between the short-ranged strong force and the much weaker, but infinite range electric force.