## Chapter 15: Fluid Mechanics Dynamics

### Using Pascal's Law

**Example 15.1 The hydraulic lift** A hydraulic lift consists of a small diameter piston of radius 5 cm, and a large diameter piston of radius 15 cm. How much force must be exerted on the small diameter piston in order to support the weight of a car at 13,300 N ?
The pressure $(F/A)$ on both sides of the hydraulic lift must be the same at the same height $y$. This lead to

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Longrightarrow F_1 = F_2\Big(\frac{A_1}{A_2}\Big)$$

$$F_1 = 13,300\Big(\frac{\pi(0.05)^2}{\pi(0.15)^2}\Big) = 1.48 \text{ x } 10^3 \text{ N}$$

There is a factor of 9 gain in lifting power by means of the hydraulic press. The same force multiplication occurs in the braking system of cars which use brake fluid to transmit the force from the brake pedal.

---

**Second Example, Variation of Pressure with Depth**
Calculate the pressure at an ocean depth of 1000 m, using the density of water as 1.0 x $10^3$ kg/m$^3$.
From Eq. 15.4 we have

$$P(y = 1000) = \rho g(1000) + 1.01 \text{ x } 10^5 = 9.90 \text{ x } 10^6 \text{ Pa}$$

This pressure is 100 times that of normal atmospheric pressure. Now you know why submarines don't have portholes.

## Buoyant Forces and Archimedes' Principle

To continue the swimming pool line of reasoning, many people are able to float in water. This is an example of **buoyancy**, the fact that objects immersed in water weigh less (or nothing) compared to what they weigh out of water. **Archimedes' principle states:**

> *Any object completely or partially immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the volume occupied by the object.*

---

**Example 15.3, page 519** A piece of aluminum ($\rho = 2.7 \text{ x } 10^3 \text{ kg/m}^3$) with a mass of 1.0 kg is completely submerged in a container of water. What is the apparent weight of the aluminum ?
The normal weight of the aluminum would be $W = mg = 1.0 \cdot 9.8 = 9.8$ N. When immersed in water, part of that weight is counteracted by the upward buoyant force of the water, $B$:

$$B = \rho_{water} \cdot g \cdot V_{aluminum} = \rho_{water} \cdot g \cdot \left(\frac{m_{aluminum}}{\rho_{aluminum}}\right) = 1 \text{ x } 10^3 \cdot 9.8 \cdot \left(\frac{1.0}{2.7}\right)$$

$$B = 3.63 \Longrightarrow T_{\text{apparent weight}} = W - B = 6.17 \text{ Newtons}$$

## Fluid Dynamics: The Equation of Continuity and Bernoulli's Equation

For moving *incompressible fluids* there are two important laws of fluid dynamics: 1) **The Equation of Continuity**, and 2) Bernoulli's Equation. These you have to know, and know how to use to solve problems.

---

**The Equation of Continuity** derives directly from the incompressible nature of the fluid. Suppose you have a pipe filled with a moving fluid. If you want to compute the amount of *mass* moving by a point in the pipe, all you need to know is the density $\rho$ of the fluid, the cross sectional area $A$ of the pipe, and the velocity $v$ of the fluid. Then the mass flow is given by $\rho \cdot A \cdot v$ because

$$\rho \cdot A \cdot v = \rho \cdot A \cdot \frac{\Delta x}{\Delta t} = \frac{\rho \Delta V}{\Delta t} = \frac{\Delta m}{\Delta t} \quad \text{(the "mass flow")}$$

If the fluid is truly incompressible, then the mass flow is the same at all points in the pipe, and the density is the same at all points in the pipe:

$$\rho A_1 v_1 = 0 \rho A_2 v_2 \Longrightarrow A_1 v_1 = A_2 v_2 \quad \text{(the equation of continuity)} \qquad (15.7)$$

---

**Bernoulli's Equation** is very powerful equation for moving, incompressible fluids, and can be derived using the conservation of Mechanical Energy (see page 459). The Bernoulli's Equation states that if you have a fluid moving in a pipe at point 1 with pressure $P_1$, speed $v_1$, and height $y_1$, and the fluid moves to point 2 with pressure $P_2$, speed $v_2$, and height $y_2$, then these six quantities are related as follows

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 \quad \text{(Energy Conservation)}$$

$$P + \frac{1}{2}\rho v^2 + \rho g y = \textbf{ constant} \qquad \text{(Alternate form)} \qquad (15.12)$$

## Using Bernoulli's Equation: The Venturi Tube and Torricelli's Law

---

**Example 15.7: The Venturi Tube** A horizontal pipe with a constriction is called a *Venturi Tube* and is used to measure flow velocities by measuring the pressure at two different cross sectional areas of the pipe. Given two pressures $P_1$ and $P_2$ where the areas are $A_2$ and $A_1$ respectively, determine the flow velocity at point 2 in terms of these quantities and the fluid density $\rho$.
First use Bernoulli's law, and take the heights $y_1 = y_2 = 0$:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Now substitute for one of the velocities, $v_1$, by using the continuity equation:

$$A_1 v_1 = A_2 v_2 \Longrightarrow v_1 = \frac{A_2}{A_1} v_2$$

$$\Longrightarrow P_1 + \frac{1}{2}\rho\left(\frac{A_2}{A_1} v_2\right)^2 = P_2 + \frac{1}{2}\rho v_2^2 \qquad \Longrightarrow v_2 = A_1\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

---

**Example Torricelli's Law (speed of efflux)** A tank with a surface pressure $P$ (at point 2) and a surface area $A_2$ has a small hole of area $A_1 << A_2$ at a distance of $h$ below the surface. What is the velocity of the escaping fluid which has density $\rho$ ?

$$P_2 + \frac{1}{2}\rho v_2^2 + \rho g h = P_1 + \frac{1}{2}\rho v_1^2 \qquad \text{and} \qquad v_2 = \frac{A_1}{A_2} v_1 \Longrightarrow v_2 \approx 0$$

$$P + \rho g h = P_a + \frac{1}{2}\rho v_1^2 \Longrightarrow v_1 = \sqrt{\frac{2(P - P_a)}{\rho} + 2gh}$$

$$v_1 = \sqrt{2gh} \quad (\text{if } P = P_a)$$

## Using Bernoulli's Law

A *large* storage tank filled with water develops a *small* hole in its side at a point 16 m below the water level. If the rate of flow from the leak is 2.5 x $10^{-3}$ $m^3$/min, determine a) the speed at which the water leaves the hole, and b) the diameter of the hole.

We assume that the tank and the hole are both open to the atmosphere. Call the top position 1 and the point of the hole position 2. So $P_1 = P_2 = P_a$. We now write Bernoulli's law:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

The *continuity equation* allows us to relate the speeds to the areas at the two positions

$$v_1 A_1 = v_2 A_2 \Longrightarrow v_1 = \frac{A_2}{A_1} v_2$$

Because the area $A_1 \gg A_2$ we can ignore $v_1$ in comparison with $v_2$ ($v_1 \ll v_2$) Now substitute $v_1 = 0$ and cancel out the equal pressures in Bernoulli's law to get

$$\rho g y_1 = \frac{1}{2}\rho v_2^2 + \rho g y_2 \Longrightarrow v_2^2 = 2g(y_1 - y_2)$$

$$v_2 = \sqrt{2gh} = \sqrt{2 \cdot 9.8 \cdot 16} = 17.7 \text{ m/s}$$

For part b) we know that the volume flow rate is the product of the area of the hole and the velocity

$$\text{flow rate} = Av$$

We first convert the flow rate given in $m^3$/minute into $m^3$/second by dividing by 60. This gives 4.167 x $10^{-5}$ $m^3$/second

$$4.167 \text{ x } 10^{-5} = A_2 v_2 = A_2 \cdot 17.7 \Longrightarrow A_2 = .2354 \text{ x } 10^{-6} \text{ m}^2$$

This is equivalent to a diameter of 0.0017 meters.