This is a closed book test. You may use a 1 page, handwritten set of notes but no texts.

Put a box around your answers, so they are visible.

You may write on the front and back of each sheet, but please keep all your work in one place.

Show all your work. Partial credit will be given. Answers that appear out of thin air will not receive credit.

This test consists of 4 questions and 7 pages (including this one).

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1. Two rectilinear coordinate systems are shown below. Find the transformation matrix that takes you from the unprimed system to the primed system. Note: $x'_3$ lies in the $x_1 - x_3$ plane and makes a 30 degree angle with the $x_3$-axis, and $x'_2$ is parallel to the $x_2$-axis. Find this matrix by finding the direction cosines between all three primed axes and the three unprimed axes.
2. The terminal speed of a baseball in free fall is 30 m/s. Assume linear drag: $F_{\text{drag}} = -b \cdot v$. If I use a baseball in an oscillator (say as a mass on a spring) what happens to the oscillation frequency? Specifically, assume the natural frequency of oscillation is:

$$\omega_0 = 2\pi \text{ rads/s}$$

(a) What is the period $T$ of oscillation for this oscillator given the air drag on the ball? Express your answer in seconds.

(b) Find the difference between this period and the period for undamped oscillations.
3. The Morse potential $U(x)$ approximates the vibrational potential energy $E$ of a diatomic molecule:

$$U(x) = U_0[1 - e^{-(x-x_0)/\delta}]^2 - U_0$$

(a) Show that $U$ has a stable equilibrium point.
(b) Find the value of $x$ for this point.
(c) What is the value of $U(x)$ at this point?
(d) Find $\omega$ of small oscillations about this point.
Extra space to work problem 3.
4. Consider a river with a current whose velocity $v$ is maximum at the center and drops to zero at either bank:

$$\vec{v} = -v_0 \left(1 - \frac{x^2}{a^2}\right) \hat{j}$$

The width of the river is $2a$, and the coordinates are such that the y-axis points upstream and the x-axis points across the stream from left bank to right bank.

Suppose that a barge in the stream is hauled around the path shown (a square of side $l$) by winches on the banks. The barge is pulled slowly and we shall assume that the force exerted on it by the current is $F_{\text{drag}} = -b \cdot v$, where $b$ is a constant. The barge is effectively in equilibrium, so that the force exerted by the winches is equal to $F_{\text{drag}}$.

(a) Is this drag force conservative?

(b) Find the total work done by the winches as they pull the barge around the path shown.

(c) Are these two results consistent? Should they be? Discuss.
Extra space to work problem 4.