Name:

- This is a **closed book** test. You may use a 1 page, handwritten set of notes but no texts.
- Put a box around your answers, so they are visible.
- You may write on the **front and back** of each sheet, but please keep all your work in one place.
- Show all your work. Partial credit will be given. Answers that appear out of thin air will not receive credit.
- This test consists of 3 questions and 7 pages (including this one).

<table>
<thead>
<tr>
<th></th>
<th>/10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>/30</strong></td>
</tr>
</tbody>
</table>
1. Consider a thin uniform ring of mass $M$ and radius $R$.
   a) Find the gravitational potential $\Phi$ at a point on the central axis of the ring. The point is a height $z$ above the center of the ring.
   b) Find the potential energy of a test mass $m$ at this point.
   c) Find the force (magnitude and direction) of the ring on the test mass $m$ in part (b). You should be able to use your answer(s) from above.
Extra space to work problem 1.
2. A simple pendulum is shown. The string has length $L$ and the weight has mass $m$.
   
a) Find the tension in the string as a function of $\theta$ and $\dot{\theta}$ (and other constants such as $m$ and/or $g$). Use the Lagrange multiplier method.
   
b) Use Newton’s Laws and a Free-Body Diagram to find the answer to part (a).
Extra space to work problem 2.
3. A uniform disk of mass $M$ and radius $R$ rotates freely about a central axle. One end of a spring of constant $k$ is attached to the axle; the other end of the spring is attached to a wall. The disk rolls without slipping. The moment of inertia of the disk is $\frac{1}{2} MR^2$.

a) Use the Lagrangian to find the equation(s) of motion of the system. You do not have to solve these equation(s).

b) What is the angular frequency $\omega$ of oscillations of the system?
Extra space to work problem 3.