Neutrino Oscillations: An Overview

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The standard model

3.0 MeV  1.2 GeV  174 GeV

6.7 MeV  117 MeV  4.2 GeV

< 3.0 eV  < 190 keV  < 18 MeV

0.5 MeV  106 MeV  1.8 GeV
Oscillations (i)

- Mixing matrix relating mass eigenstates and flavor states results in oscillation

Standard parameterization (3-ν):

\[
U = \begin{pmatrix}
    c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\
    -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i \delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta} & s_{23} c_{13} \\
    s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i \delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i \delta} & c_{23} c_{13}
\end{pmatrix}
\]

where \( s_{jk} = \sin(\theta_{jk}) \), \( c_{jk} = \cos(\theta_{jk}) \).
Oscillations (ii)

- Using quantum theory, we may determine the (ultra-relativistic) oscillation probability for ν of energy E and source-detector distance L.

\[
P_{\alpha \rightarrow \beta}(L/E) = \text{tr} \left[ e^{i\frac{ML}{2E}} P^\alpha e^{-i\frac{ML}{2E}} P^\beta \right]
\]

where \( M = \text{diag}(m_1^2, m_2^2, m_3^2) \)

and \( (P^\alpha)_{jk} = U^*_\alpha j U_{\alpha k} \).
Oscillations (iii)

- Or in more familiar terms...

\[
P_{\alpha \to \beta} \left( \frac{L}{E} \right) = \delta_{\alpha \beta} - 4 \sum_{j<k}^{3} \Re \left| U_{\alpha j} U_{\alpha k}^* U_{\beta k} U_{\beta j}^* \right| \sin^2 (\phi_{jk}) 
+ 2 \sum_{j<k}^{3} \Im \left| U_{\alpha j} U_{\alpha k}^* U_{\beta k} U_{\beta j}^* \right| \sin (2 \phi_{jk})
\]

where \( \phi_{jk} = \Delta_{jk} \frac{L}{4E} \)

and \( \Delta_{jk} = m_j^2 - m_k^2 \).
Present parameter values

\[
\theta_{12} = 0.57 \pm 0.06
\]

\[
0 \leq \theta_{13} \leq 0.23
\]

\[
\theta_{23} = 0.78 \pm 0.17
\]

\[
\Delta_{21} = \left( 7.1^{+1.8}_{-1.1} \right) \times 10^{-5} \text{eV}^2
\]

\[
\Delta_{31} = \pm \left( 2.0^{+1.2}_{-0.8} \right) \times 10^{-3} \text{eV}^2
\]

\[
\delta = ??
\]

Experimental overview

- Solar (e.g., Homestake, SAGE, Kamiokande, SNO)
- Atmospheric (e.g., Super-K)
- Reactor (e.g., CHOOZ, KamLAND)
- Accelerator beam-stop (e.g., LSND, K2K)
Solar neutrinos

\[ L \sim 10^{11} \text{ m} \]

E and the expected \( \nu_e \) flux can be got from the SSM

\[ \frac{L}{E} \sim 10^{10} \text{ m/MeV} \]
Solar neutrino deficit (i)

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Data/Theory</th>
<th>Reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAGE</td>
<td>0.54 ± 0.05</td>
<td>$\nu_e + ^{71}\text{Ga} \rightarrow ^{71}\text{Ge} + e^-$</td>
</tr>
<tr>
<td>GALLEX</td>
<td>0.61 ± 0.06</td>
<td>$\nu_e + ^{37}\text{Cl} \rightarrow ^{37}\text{Ar} + e^-$</td>
</tr>
<tr>
<td>GNO</td>
<td>0.51 ± 0.08</td>
<td>$\nu + e^- \rightarrow \nu + e^-$</td>
</tr>
<tr>
<td>Homestake</td>
<td>0.34 ± 0.03</td>
<td></td>
</tr>
<tr>
<td>Kamiokande</td>
<td>0.55 ± 0.08</td>
<td></td>
</tr>
<tr>
<td>Super-K</td>
<td>0.465 ± 0.005</td>
<td></td>
</tr>
</tbody>
</table>

Note: SK is sensitive mostly to $\nu_e$ - but also to $\nu_{\mu\tau}$ (down by factor 7)
SNO results

- Elastic scattering $R = 0.47 \pm 0.05$
- Charged current $R = 0.35 \pm 0.02$
- Neutral current $R = 1.01 \pm 0.13$

\[
\begin{align*}
\nu + e^- &\rightarrow \nu + e^- \\
\nu_e + d &\rightarrow p + p + e^- \\
\nu + d &\rightarrow p + n + e^-
\end{align*}
\]

\[
\begin{align*}
\nu_e \quad \text{flux} &\quad = 1.76 \pm 0.05 \pm 0.09 \\
\nu_{\mu\tau} \quad \text{flux} &\quad = 3.41 \pm 0.45 \pm 0.46 \\
\text{SSM flux} &\quad = 5.05 + 1.01 - 0.81
\end{align*}
\]
Atmospheric neutrinos

Incident cosmic rays produce $\pi^\pm$ which decay

$$\pi^\pm \rightarrow \mu^\pm + \nu_\mu (\bar{\nu}_\mu) \quad \mu^\pm \rightarrow e^\pm + \nu_e (\bar{\nu}_e) + \bar{\nu}_\mu (\nu_\mu)$$

At Super-K:

$$L \sim 10^3 - 10^7 \text{ m} \quad E \sim 10^3 \text{ MeV}$$

$$L/E \sim 10^0 - 10^4 \text{ m/MeV}$$
Super-K results

The measured/expected ratio for $\mu$ events shows dependence on $L/E$. 
Reactor neutrinos

Measure $\bar{\nu}_e$ oscillation probability for $E \sim 3$ MeV at a fixed $L$

CHOOZ ($L \sim 10^3$ m) saw no deficit

KamLAND ($\langle L \rangle \sim 10^5$ m) does!
KamLAND results

Measured / expected = \(0.658 \pm 0.044 \pm 0.047\)
Beam-stop neutrinos

K2K = KEK to Super-K

Measure $\nu_\mu \ (E \sim 10^3 \text{MeV})$ with a long baseline $L \sim 10^5 \text{m}$.

Measured / expected = $0.55 \pm 0.19$
Our analysis

- Construct a model of the experiments assuming CP is conserved
- Explore the acceptable region for $\theta_{13}$

NB: We bound the mixing angles as below

$$\theta_{12} \in [0, \pi/2], \quad \theta_{13} \in [-\pi/2, \pi/2], \quad \theta_{23} \in [0, \pi/2].$$
Allowed region

$\Delta \chi^2 \ vs. \ \theta_{13}$

95% CL
90% CL
Comments

- Within our model, we find that $\theta_{13}$ lies between -0.17 and 0.24 at the level of 1-\(\sigma\).

- In a perturbative expansion about $\theta_{13}$ and the ratio $\Delta_{21}/\Delta_{31}$, terms linear in $\theta_{13}$ are suppressed by the mass ratio $\sim 0.03$.

- Is there a region in which this suppression can be overcome so that the positive and negative regions for $\theta_{13}$ might be experimentally distinguished?
Best guess

- Consider a region where the $\Delta_{31}$ oscillations are incoherent [i.e., $\langle \sin^2 \phi_{31} \rangle = 1/2$], but the $\Delta_{21}$ oscillations are coherent.
- Examine $P_{e\mu}$, $P_{\mu\mu}$, $P_{\mu\tau}$ oscillation channels.
- The sign of the mixing angle has greatest impact whenever $\sin^2 \phi_{21}$ is maximal; i.e., look in the region:

$$L/E \sim 1.6 \times 10^4 \text{ m}/\text{MeV}.$$
Size of the effect

Using the 1-$\sigma$ bounds: $\theta_{13}^+ = 0.24, \quad \theta_{13}^- = -0.17$.

\[
\frac{P_{\mu\mu}(\theta_{13}^+) - P_{\mu\mu}(\theta_{13}^-)}{P_{\mu\mu}(\theta_{13}^+) + P_{\mu\mu}(\theta_{13}^-)} = -0.25, \quad \frac{P_{e\mu}(\theta_{13}^+) - P_{e\mu}(\theta_{13}^-)}{P_{e\mu}(\theta_{13}^+) + P_{e\mu}(\theta_{13}^-)} = 0.18.
\]

The $P_{\mu\tau}$ channel exhibits little dependence on the sign of $\theta_{13}$, as $\theta_{23}$ is nearly maximal.
Plot of $P_{\mu\mu}$ in this region

$\theta_{13} = -0.2$
$\theta_{13} = 0$
$\theta_{13} = 0.2$
Plot of $P_{e\mu}$ in this region

$\theta_{13} = 0.2$

$\theta_{13} = 0$

$\theta_{13} = -0.2$
Plot of $P_{ee}$ in this region

\[ \theta_{13} = 0 \]
\[ \theta_{13} = \pm 0.2 \]
$P_{\alpha\beta}$ as a function of $\theta_{13}$

$(L/E \sim 1.6 \times 10^4 \text{ m/MeV})$
Caveat

• The mixing angles $\theta_{13}$ and $\theta_{23}$ are correlated.
• Future work: Examine this correlation and its implications....